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for calculating machine

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A TREATISE

ON

SCREW PROPELLERS

AND THEIR

STEAM-ENGINES.

A TREATISE

ON

SCREW PROPELLERS

AND THEIR

STEAM-ENGINES,

WITH

PRACTICAL RULES AND EXAMPLES HOW TO CALCULATE
AND CONSTRUCT THE SAME FOR ANY
DESCRIPTION OF VESSELS,

ACCOMPANIED WITH

A TREATISE ON BODIES IN MOTION IN FLUID,

EXEMPLIFIED FOR PROPELLERS AND VESSELS;

ALSO,

A FULL DESCRIPTION OF A CALCULATING MACHINE.

BY

J. W. NYSTROM.

PHILADELPHIA:
HENRY CAREY BAIRD,
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PREFACE.

In this treatise on screw-propellers and the engines employed to drive them, the Author offers the results of several years' experience and observation respecting their performances.

One of the objects aimed at has been to obtain formulæ to follow the variations that arise in practice; which formulæ are here introduced and exemplified.

Theory and philosophy have been followed as far as they correspond with results; but, when they were found to differ, proper co-efficients were introduced to make the formulæ simple and practical. The difference between theory and practice may have arisen from circumstances not fully understood, and which, if entered into, might complicate the calculations with no remuneration.

Algebraical formulæ may appear difficult to those not used to their application; but, when they are accompanied by examples, almost any one can insert numerical values and perform the computation. Algebra is becoming every day more popular, and will soon be a stranger to none who are at all interested in calculations.

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The Author would apologize for the manner in which the work is written, which is partly owing to his not being perfect in the English language; but, even in his native tongue (Swedish), possessing no merit as a writer; but he claims *originality* in the scientific part.

The *Treatise on Bodies in Motion in Fluids* is not what the Author desired to make it; but, without proper experiments expressly for the subject, it could not be materially improved. However, the scientific part of the matter is a fair field for the investigator.

In the description of the calculating machine, are introduced matters, with examples, not always accessible to persons who frequently feel the want of them.

CONTENTS.

TABLES.

PLATES.

ERRATA.

Page 56, line 1,	*for* "Strengthen,"	*read*	Straighten.
" 66, " 7,	" "be L in,"	"	be in.
" 76, lines 9 and 10,	" "18,"	"	0.55.
" 80, line 29,	" "18,"	"	0.55.
" 81, " 9,	" "inverse,"	"	immerse.
" 131, " 17,	" "a,"	"	a.
" 132, " 24,	" "a. $sin.\phi$,"	"	$a\ sin.\phi$.
" 135, " 14,	" "a,"	"	a.
" 136, " 10,	" "a,"	"	a.
" 136, " 14,	" "a,"	"	a.
" 166, " 9,	" "$\phi = 6°.55.6$,"	"	$\phi = 6° 38.4'$.
" 201, " 15,	" "$b\ sin.B$,"	"	$c\ sin.B$.

On page 27, the pressure p means the *force* in the direction of the connecting-rod; or the pressure on the steam piston multiplied by $sec.w$. This makes a slight difference in the momentum and friction.

INTRODUCTION.

PROPELLER.—By the term *propeller*, we mean an instrument which is used in navigation for the propelling of vessels. It has a centre-line from which a number of blades extend to a circumference, in which the aforesaid line is the centre. The propeller is generally fitted in the stern of the vessel, so that its centre-line is parallel with the length of the vessel, and wholly below the surface of the water.

PROPELLER-BLADES.—The *blades* have the form of a helixoidical surface of the thread of a screw, so that each blade forms a separate thread, and, therefore, bears the name of *screw-propeller*. Its mode of action is such that, when placed in the stern of a vessel in water, and revolved in the same, the water within the circuit of the propeller acts as a *nut* for the *screw*, and the propeller screws the vessel in the direction of its centre-line.

PITCH.—The *pitch* of a screw is the space which the screw moves in its *nut* while turning it round once (supposing the nut to be stationary). If the water in which the propeller acts was a stationary nut, when the propeller makes one turn the vessel should move a space equal to the *pitch* of the propeller; but, as the water is a free movable body, it will also move a space, so that the two motions of the water and vessel together will be equal to the motion of the pitch in a unit of time. Those two motions are termed *advance* and *slip.*

Advance is the motion of the vessel forwards.

Slip is the motion of the water backwards.

The *advance* and *slip* are measured by the *pitch* as a unit. Suppose the *pitch* of a propeller was 100 inches, then, when the propeller makes one revolution, the vessel should *advance* 100 inches if there was no *slip;* but, if the vessel is found to advance only 75 inches, then the slip is 25 inches. This slip is generally noted in *per cent.* of the *pitch.*

A TREATISE

ON

SCREW-PROPELLERS AND THEIR STEAM-ENGINES.

SUGGESTIONS FOR EXPERIMENTS ON SCREW-PROPELLERS AND THEIR STEAM-ENGINES.

A QUESTION of considerable interest at the present time is the relative merits of screw-propellers and paddle-wheels; which of them is the most available and possesses the greatest superiority for navigation. Were we to decide from the present aspect of public opinion, we would be almost compelled to say screw-propellers are on the wane. But, perhaps, good reasons might be assigned for this seeming loss of popularity, without detracting from the merit of the screw-propeller. Compared with paddle-wheels, if we put on more steam or power the vessel will run faster; but, for screw-propellers, there is a more delicate arrangement between the steam and the action in the water; that if you say, "Put on high steam; it must run," you will be disappointed. Then, is preeminence justly on the side of the paddle-wheels, with the same amount of fuel? As the screw-propeller is the most suitable and valuable instrument in navigation, the question is worthy of attention, and can be answered only by a series of patient and impartial experiments, extending through several different vessels and propellers.

Experiments, we know, have been made, again and again, but the principal feature has been too often overlooked.

In these suggestions, we will demonstrate the principal features in the following questions.

THE PROPELLER.

1. *Pitch.*—On what does the pitch depend? If on the *slip* and *diameter* of the propeller, and of the arrangement of the steam-engine, what will the proper *pitch* be when those quantities are given? Or what will be the difference with more or less pitch to the economy of power?

English engineers take a very narrow pitch in proportion to the diameter—from 1¼ to 1½ the diameter. This, we think, results from experiments and observations of the following two facts:—

a. That less pitch gives less slip. But, if the slip is no measure of loss of effect, the slip caused by different pitch should not govern the pitch, as the slip caused by the resistant area of the vessel to the acting area of the propeller when brought into action. (See further about those areas.)

b. That less pitch does better in head winds. But, if any proper arrangement with more pitch could balance the former, it will have a greater advantage in calm and fair weather. (See further.)

2. *Advantage and disadvantage with more or less pitch.*—How will fair and head winds affect more or less pitch in propellers? It is a fact that propellers

with more pitch and slip employ the effect better for propelling in still water or fair winds; but, in head winds, propellers with less pitch and slip will do best, owing to the slip of the more-pitched one;

exceeds $$S=\frac{200}{Pn}\sqrt{D},$$

and a portion of the power is expended in agitating the water. Then the question comes, is this formula correct? and how will it affect the coefficient 200, if the propeller is centripetal, as described in these pages? For vessels which are exposed to much head wind, a less-pitched propeller is preferable; but if a well-proportioned centripetal propeller with more pitch could balance the less-pitched one, in those circumstances it should be preferred, because, when the vessel has a fair wind, and the sails are set, the less-pitched propeller is often of no use for propelling, and sometimes has what is called a *negative slip*, while a more-pitched one would still act to propel the vessel with great effect. From this we come to the conclusion, that a propeller in the Pacific Ocean should have more pitch than one in the Atlantic Ocean; because in the latter there is greater liability to storms than in the former. Propellers running between San Francisco and China should be more pitched.

Inventions have been patented, both in America and Europe, to change the pitch of propellers, while in the water, by some arrangement of cog-wheels at the hub to twist the blades at pleasure.

3. *Slip.*—On what does the slip depend? If on the diameter and pitch of the propeller, brought into action with the greatest immerse section area, form, and friction area of the displacement, what will the slip be when those quantities are given?

4. *Loss of effect by slip.*—Is the slip a correct measure of loss of effect or quality of the propeller, as thus far has been a generally-received opinion? We think experiments will prove that more slip will be more economy of power until a certain limit, depending upon the proper arrangement, and that the slip is merely a measure of the power which propels the vessel.

5. *Acting area of the propeller.*—On what does the acting area of the propeller depend? If on the pitch and diameter, what will it be when those quantities are given? The acting area means a plane at right angle to the centre line of motion; and having the same velocity as the slip, will sustain a resistance equal to that of the propeller, in the direction of motion parallel to its centre line, at the given immersion.

6. *Resistant area of the vessel.*—On what does the resistant area of the vessel depend? If on the greatest immerse section area, form, and friction area of the displacement, what will the resistant area be when those quantities are given? The resistant area of a vessel means a plane at right angle to the direction of motion, and, having the same velocity as the vessel, will sustain a resistance equal to that of the vessel.

7. *Velocity and resistance to the acting and resistant areas.*—Is the resistant to those areas in proportion as the square of their velocity? If so, or whatever the proportion may be, when brought into action it determines the slip of the propeller and velocity of the vessel.

It is evident that the same propeller must have the same acting area, independent of its velocity, and the same must be the case with the resistant area of vessels; but it is found that, for propellers, this resistant does *not* increase as the square of its velocity, and it is probable that it will come nearer that proportion as the pitch is less in proportion to diameter. The resistant area of the vessel contains the two quantities, *greatest immerse section area* multiplied by the *sine* for its angle of resistance; and the *friction area.* The resistance to the former is, as the *square of its velocity*, but to the latter more *direct as the velocity.* Therefore, the total resistance will be nearer the square of its velocity, as the friction area is less in proportion to the greatest immerse section area.

8. *Velocity of the propeller.*—On what does the proper velocity of a propeller depend? If on the pitch, slip, and diameter of the same, what will the proper velocity be when those quantities are given?

In regard to the proper velocity, it is known from experience, especially from canal and tow-boats, that, when the velocity of the propeller increases, the speed of the vessel will also increase until a certain limit;

when exceeding that, the speed of the vessel will not increase, which may be for the following reasons:—

a. When the propeller exceeds a certain velocity, the hydrostatic pressure of the water is not sufficient to supply solid water into the circuit of the propeller. This can be partly overcome by giving the propeller an expanding pitch in two directions, so that it expand from the *fore* to the *after edge;* and, in the same proportion, expand *from the centre to the periphery.* The former expansion is measured by the angles v v', and the latter by the angles $w°$ w' w'. (See Plates V. and VII.)

b. The centrifugal force of the water acts to agitate itself, in proportion as the square of the velocity of the propeller. This has led to the suggestion of propellers with curved blades, so proportioned to the centrifugal force that, at any distance from the centre within the propeller, the water will obtain an helixoidical motion backwards, parallel to the centre line of the propeller. It is *not theory* that has led to this idea, but actual practice, by observation of the instrument's operation in the water, of which there has been a very good opportunity of judging. The results of the observations have been worked out by theory, as will be seen in these pages.

9. *Centripetal propeller.*—Is there any advantage in the blades being curved? And, if so, what does the curve depend upon? If on the pitch, slip, and velocity, what will be the curvature when those quantities are given?

It is evident that the curve should be an arithmetic spiral; but it is probable that the pitch of this spiral should be independent of the slip or velocity of the propeller for the following reason:—

When the generatrix for a screw is an arithmetic spiral drawn on a plane at right angle to its axis, it is the same as if the generatrix for the screw was a straight line with an inclination $= U$ to the axis. This angle U will be found by the formulæ

$$tang.\ U = \frac{180\ D}{w^\circ\ P}, \quad \ldots\ldots \quad (1)$$

in which the letters denote

$P =$ pitch of the propeller, and

$D =$ diameter,

$w^\circ =$ the angle w° in degrees. (See Plate VI. & VII.)

When this inclination $U = 45^\circ$, the centrifugal force acts with its greatest advantage to propel the vessel, independent of the slip or velocity of the propeller. Then $tang.\ U = tang.\ 45^\circ = 1$, and

$$w^\circ = \frac{180\ D}{P}. \quad \ldots\ldots \quad (2)$$

A propeller constructed on this principle will be seen hereafter, with its calculations.

10. *Expanding pitch.*—Is there any advantage in the expanding pitch, and on what does the expansion depend? If on the slip, what will it be when the slip is given? The expanding pitch changes the inclined generatrix to a curved line, drawn on the same plane

as the axis for the screw, and momentarily changes its form.

11. *Length of the propeller.*—On what does the length of the propeller depend? If on the number of blades, and proportions of pitch, diameter, and slip, what will be the proper length when those quantities are given? The length is of some importance for the strength and room in the stern of the vessel.

12. *Number of blades.*—On what do the number of blades depend? If particularly *on the slip,* and proportions of pitch and diameter, what will be the proper number of blades when those quantities are given?

It is very well known that propellers with two or three blades will do as well, or better than those with four or five blades, but in those comparisons the quantities on which the number of blades depend are often neglected. We do not mean to say that a more-bladed propeller will do better because there are more blades; in that case, there is no hope (except when the slip is very slight); but, if we can make it do as well as the less-bladed, we will gain the following advantages:—

a. The more-bladed propeller occupies less space in the stern of the vessel, because it is shorter than the less-bladed, if well proportioned; and the vessel can be made neater and stronger in the stern, especially when made of wood.

b. The more-bladed propeller is stronger and safer, and does not shake the vessel as much as the less-bladed, which increases the comfort of the same, and makes it more durable and safe.

c. When the vessel is running in a hard sea, the waves will lift up the stern of the same; so that often more than one-half of the propeller will be above the water; then, if it is a two-bladed propeller, it can happen that both the blades come over the water, which causes a violent shock in the steam-engine, and in danger of losing the propeller-blades. This has caused Englishmen to apply governors to the engines where the two-bladed propeller is used; but even that will not fully answer the purpose. If the propeller has three or four blades, there must be at least one blade in the water, but then the whole power from the steam-engine is acting upon that blade, which then stands in danger of breaking, and, if it breaks, it is probable that the others will follow. When the propeller has five blades, there must be at least two blades in the water; but those blades are smaller in proportion to the three-bladed one, which still more increases the safety of the five-bladed one, and, if supported by two narrow bands (as shown in Plate VII.), when only two blades act in the water, the resistance will still act on the other three blades through the bands, and render the five-bladed propeller perfectly safe.

In the above statement of number of blades, there must be an exception when the propeller has a very narrow pitch in proportion to the diameter—as the

English propellers; but such a proportion cannot be adapted to direct-action steam-engines.

d. When the vessel is to run by sail, and the propeller is coupled off from the steam-engine, and runs by the resistance of the water caused by the speed of the vessel, this resistance is greater to a propeller of less pitch and number of blades, and has caused another arrangement in the stern of the vessel to hoist up the propeller when the vessel is to run by sail. This complicated mechanism increases the expense, and spoils the stern of the vessel, and is, after all, nothing to depend upon. If the coupling can be made strong enough, it will be clumsy in its place; and, in case of war, the vessel might come in an unsafe, turbid, and shallow water, where some obstacle would fasten in the coupling and prevent the proper manœuvring. In a propeller with more pitch and blades, and a narrow length, the resistance will be so trifling that the propeller can, without detriment, remain in its proper place.

The disadvantage with the more-bladed propeller is when first starting the vessel, as it does not give so quick starting as the less-bladed, owing to the slip, but, when started to its full speed, there will be no difference in their performance, if properly arranged.

13. *Resistance of the propeller.*—What will be the resistance of the propeller, when the vessel is running by sail and no steam, under the following circumstances?

a. The propeller standing stationary in its place to the vessel.

b. Coupled loose from the steam-engine, and can revolve freely.

c. What will be the difference in the resistance when the propeller has more or less pitch and number of blades, remains in its place, and hoists up?

d. Will the resistance be less when the propeller is centripetal? which we think it will be, owing to a disadvantage by such a propeller for backing when worked by steam.

14. *Regular screw.*—Shall the propeller be a regular screw? This question will certainly be answered, first by mathematicians, that *it shall be a regular screw.* It is evident that, if there was no slip, the propeller should evidently be a regular screw, but, as such an instance never can exist in water, the propeller shall accordingly *never be a regular screw.* If the irregularity depends on the slip, what will it be when the slip is given?

The question may be asked, if the propeller can be a screw at all if it is *not regular?* In the answer to that, we have three different propellers, viz.:—

a. Regular screw, is a propeller which has a uniform pitch on all its helixoidical surface.

b. Irregular screw, is the propeller which has an expanding pitch, in one or two directions, so that the generatrix for the screw runs through the centre of the propeller.

c. No screw at all, is the propeller where the propeller-blades form an angle with the centre line in the centre of the propeller, and the generatrix for each propeller-blade has a centre line *not* common to the propeller or to themselves.

The irregular screw is the one which should be adopted for propelling in fluids. Frenchmen have come to the result, from experiments, that the more the propeller-blades are cut out at the hub, the more effective is the propeller. In those experiments, the propellers have certainly been regular screws. If a propeller has an expanding pitch, in the direction of the radix, proportioned to the slip, the cutting off the blades at the hub would show no difference.

15. What power is required to give propellers a certain number of revolutions per minute, when the diameter, pitch, and slip are given? In what proportion does the power differ with the pitch and slip?

THE STEAM-ENGINE.

The steam-engine is one of the most valuable agents in navigation; and, as we thus far have no other propelling agent in view, which can substitute the steam, there is perhaps both time and room to improve the arrangement of the steam-engine, particularly for screw-propellers. We have for years had high expectations of the electro-magnetism as a motive power, but have hardly noticed our disappointment. If it comes *slowly*, we hope it will be *sure*, and we do not relinquish the hope that yet, in our day, it will *not always be steam navigation.* But we steam-engineers will go a-head with the steam-engine, and wish the electro-magnetic engineers a *soon and good success.* In regard to the propeller-engines, the first point which requires attention is the arrangement of the air-pumps and their valves.

To obtain a simple and compact direct-action propeller-engine, the air-pump ought to be applied direct to the cross-head or steam-piston; but then it is found that the air-pump works too fast, therefore, the air-pump is *geared* to run slower, or, perhaps, rather *gear* the propeller, and a complicated and heavy machinery will be the result.

When the engine is direct action, the air-pump is often worked by a walking-beam, with a reduced stroke of the steam-piston, and thereby obtain a slower motion of the same.

If we know the law which governs it, perhaps we can work the air-pump at any reasonable speed. It is evident that the vacuum, or, more correctly, the pressure in the condenser, is what governs the proper speed of the air-pump, and by those two the area of the air-pump valves must be proportioned.

To each pound of pressure per square inch answers a column of water about 27 inches; then, when the vacuum in the condenser is 10 pounds, the pressure in the same will be about $4\frac{1}{2}$ pounds, which answers to a column of water $4\frac{1}{2} \times 27 = 121$ inches, which is the space from which the velocity of the water through the valves is to be ascertained (deducting the friction and slip through the valves, air, &c., about 40 or 60 per cent.). 121 inches is about 10 feet, fallen through by a body will obtain a velocity of 25 feet, deducting 50 per cent., will be about $12\frac{1}{2}$ feet per second, the velocity of the water through the valves. Then the velocity of the air-pump piston is to $12\frac{1}{2}$ as the area of the valves is to the area of the air-pump. Call the area of the air-pump $= 2$, and the area of the valve $= 1$, the proper velocity of the air-pump piston will be $6\frac{1}{4}$ feet per second.

Capacity of the air-pump, and area of its valves.— What will be the proper capacity of the air-pump, when the capacity of the steam-engine, and density of the exhausted steam are given, to obtain a certain vacuum? And what will be the proper area of its valves, when the velocity of the air-pump piston and vacuum in the condenser are given? Rules have been laid down, but they are incomplete for direct-action propeller-engines, where the air-pumps are attached direct to the cross-head, and, to make such rules simple and applicable for any circumstance, they should be accompanied by tables as follows:—

For the valves, a table should be constructed so that the vacuum on the top of the columns in pounds per square inch, and the velocity of the air-pump piston in feet per second in the first left column, and where they meet, should be the coefficient for multiplying the area of the air-pump piston to obtain the area of the valves.

For the capacity of the air-pump, the vacuum should be marked in pounds per square inch on the top of the columns, and, in the first left column, the density of the exhaust steam in pounds per square inch; in the columns where these meet, should be the coefficient for multiplying the capacity of the steam-cylinder to obtain the capacity of the air-pump (when fixed a mean temperature for the injection-water). Sometimes it is used from 40 to 60 pounds of steam, and cut off the steam first at $\frac{1}{2}$ or $\frac{5}{8}$ths the stroke; then the exhaust steam will be from 20 to 30 pounds, excluding the atmosphere. For another arrangement, the exhaust steam being only 6 or 8 pounds.

This, of course, makes a considerable difference in the arrangement of the air-pump and its valves, and, when attached direct to the crosshead, it is of great importance that proper attention is paid to it.

An ingenious engineer invents some new and simple arrangement of engines, and knows he can apply the air-pump direct to the crosshead, but can obtain only a certain proportion of areas of the valves and air-pump piston, and the stroke and velocity are given: then it is of no use making the air-pump larger than is necessary to work well. By reference to the rules before spoken of, it can easily be regulated to suit the arrangement.

These rules could easily be calculated, but, to place confidence in them, it is necessary to lay them down *only from experiments*, and, in those experiments, there must be proper arrangements in order to make the rules applicable for any circumstances; which, for propeller-engines, are the most variable.

Cut off the steam.—In order to save steam, or, more correctly, to employ the effect of steam to a higher degree, it is common to shut off the admittance of steam to the steam cylinder when the piston has moved a part of the stroke. From that cut-off point, the steam acts expansively with a decreased pressure on the piston, and causes an irregularity in the momentum, by transferring the straight linear motion to the rotary. This irregularity is of no consequence where there are some heavy revolving pieces or fly-wheel to regulate the motion; but for screw-propellers it is worthy of attention, and can be regulated by giving a little more or less steam on one side of the piston, particularly where two engines are working at one or opposite cranks, which are the engines we will allude to in this work.

Rotary Motion, by Crank and Connecting-Rod.

When the straight linear motion is to be transferred to the rotary by crank and connecting-rod, there exists some irregularity in the momentum for the rotary motion.

Momentum is a force multiplied by the length of the lever on which it acts: see **Fig. 1.** The line l represents the connecting-rod from a steam-engine, attached to the crank r at d, which revolves around the centre C. p is the force from the steam-engine, acting on the lever a, which is drawn from the centre C, at right angles to the connecting-rod l. The momentum for that force will be expressed as

$$\text{Momentum} = pa.$$

Let S denote the distance which the piston has travelled when the crank stands in the angle v, and expressed in a fraction of the stroke.

$r = 1$, the length of the crank.

$b =$ distance from the centre of the rotary motion to the centre of the cross-head.

$l = 4\ r$, the connecting-rod twice the stroke we have

$$S = \frac{l + r - b}{2}, \quad . \quad . \quad . \quad . \quad (1)$$

and

$$b = \frac{l \; sin.x}{sin.v}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$sin.w = \frac{r \; sin.v}{l}, \quad . \quad . \quad . \quad . \quad (3)$$

$$a = \frac{l \; sin.x \; sin.w}{sin.v} \quad . \quad . \quad . \quad (4)$$

$$a = b \; sin.w \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$x + w + v = 180°. \quad . \quad . \quad . \quad (6)$$

Suppose we divide the circle of motion into 24 equal parts, and draw the radius. From the above formulæ calculate the values of a, b, and S. In each of those 24 parts, set off the corresponding value of a from the centre C, and join them with a curved line $m\ n$, as represented in **Fig. 2.** The values of a, b, and S, corresponding to their angles v, w, and x, are set up in the Table I. In any position of the crank, the distance from the centre to where the crank crosses the curved line $m\ n$, represents the momentum for the rotary motion, when the force $p = 1$.

When the crank stands in the position 3, the momentum for the rotary motion is a, but in the position 9, the momentum is $C\ e$. If the connecting-rod was infinite, the curved lines m, n would be two circles, with a diameter equal to the radius of the crank, but the shorter the connecting-rod is, the more it varies from a circle; when the pressure p is constant throughout the whole stroke, but when the steam is shut off at a part of the stroke, the curved line will be a more irregular one; and by calculating the decreased pressure on the piston in each 24 positions of the crank, multiplied by the corresponding value of a, is the momentum for the rotary motion. If the steam is shut off at $\frac{1}{3}$ of the stroke, it will be a curved line, C, n, o, d, **Fig. 2**, which is a clear view of the irregularity. The two inner circles represent the *mean momentum* for full steam, and cut off $\frac{1}{3}$ of the stroke, which is as 620 : 434, see column a, and ap, Table II. The column n shows the number of divisions in which the crank stands, and v, the angle of the crank to the centre line of the steam-engine. See Fig. 1 for the angles w and x.

When two engines are working at right angles on one common crank, the momentum for the rotary motion will be equal to the sum of the momentum from both the engines, and so divided, that when the momentum of one engine is 0, it is near its maximum of the other engine, and there exists no dead point.

Table and **Fig. 3.**—Set two columns of a together, so that the 6th division in one corresponds with 0 in the other, as seen in the table: the 6th division is 0.970, which, in the column a', corresponds with 0 in the column a. Add the two columns a and a' together for each division, place the sum in the column $a + a'$, which then will represent the momentum from both the engines.

Fig. 3. Draw the two lines $C\,b$ and $C\,b'$ at right angles with each other, which then will represent the centre lines of the two steam-engines; proceed, as in Fig. 2, by dividing the circle of motion into 24 equal parts, and start from o at b, in the direction of the arrow i. On each division, set off the corresponding value $a + a'$, join them as described in Fig. 2; it will be a curved line m, n, m, n. If the connecting-rods were infinite, the curved line would have been four half circles, with a radius $=$ 0.707 times the radius of the crank. In 3 and 15, the curves are nearly half circles, but in 9 and 21, it differs more, owing to the connecting-rod being shorter than infinite, or in this figure only twice the stroke. In the half circle, 15, 21, 3, the momentum is greater than in the other half circle, 3, 9, 15. By observing such a steam-engine in motion, it will be found that, when the crank passes the point o, it runs fastest, and when passing 12, it runs slowest. If there is any additional work applied to the steam-engine, as, for instance, a single acting air-pump, it should be so attached to the engine, that when the crank stands in 21, the air-pump piston should stand on half the stroke when going up.

When the steam is cut off at $\frac{1}{3}$ the stroke, the momentum line will be represented by the curve o, o, o, o, a more irregular one. The momentum is greatest and more regular in the half circle, 9, 15, 21; less and more irregular in the half circle, 21, 3, 9, which shows that a little more steam should be admitted on the top of the pistons, as to cut off the steam a $\frac{3}{8}$ of the stroke. The drawn circles represent the mean momentums, and the dotted one, the centre circle of the crank.

On page 75 is described an opposite crank, Fig. 9. If we, in this same position of the steam-engine, apply the opposite crank, the difference from Table III. will be, that the 18th division in the column a will correspond with o. (See Table IV. and **Fig. 4.**) The momentum for the rotary motion will still be $a + a$, but the friction in the bearings will be $\pm a \mp a$, represented in the column. The momentum line, $m\,n$, $m\,n$, will be precisely the same as in Fig. 3, but it has another position to the steam-engine, and the maximum momentum is at 3; that is, the crank which is attached to the engine b, stands in the third division from o, but the crank which is attached at b' stands in 15. In the four points where the momentum is greatest, the friction is o. In the bottom of the columns $a + a$ and $\pm a \mp a$, we have the mean momentum for rotation, which is to the mean momentum for friction as 1.24 : 0.527. When two engines are working at right angles on a single crank, the mean momentum for friction will be $2 \sin. \frac{1}{2} 90° = 1.414$; then we have the friction in a single crank is to the friction in a double crank as 1.414 : 0.527. When the steam is cut off at $\frac{1}{3}$ of the stroke, the momentum line for rotation will be o, o, o, o. For the friction, the momentum line will be f, f, f, f. The arrows in the friction line show the direction in which the shaft presses in the bearing. The small circle shows the mean momentum for the friction.

Fig. 5 shows the momentum lines from two steam-engines working at 120° on one single crank. When working with full steam, the mean momentum for friction will be $2 \sin. \frac{1}{2} 120° = 1.732$.

Fig. 6 shows the momentum lines from two engines working at 120° on opposite cranks. In the bottom of the column $\pm a \mp a$, we have the mean momentum for the friction $= 0.464$. Then, when two engines are working at 120°, the friction by the single crank is to the friction by the opposite crank as 1.732 : 0.464.

Fig. 7 shows the momentum line from two engines working at 150° on one crank. The momentum for the friction will be $2 \sin. \frac{1}{2} 150° = 1.9318$.

Fig. 8. Two engines working at 150° on opposite cranks. The friction in the column $\pm a \mp a = 0.285$, then, when the angle is 150°, the friction in the single crank is to the friction in the opposite crank as 1.932 : 0.285.

The loss of effect by friction with shafts of wrought-iron, in bearings of bronze, will be, in horse-power, about

$$H = \frac{D\, n\, p^*}{233000}$$

in which

D = diameter of the shaft in inches,
n = number of revolutions per minute,
p = total pressure in the bearings.

* Morin's Experiments.

In the steam-engine on Plate VIII., is calculated the values of

$p =$ 61,575 pounds pressure on the piston,

$D =$ 12 inches, diameter of the shaft,

$n =$ 48.5 revolutions per minute,

$f =$ friction coefficient, which, for this purpose, will be found in the friction columns in the accompanying tables.

Let us, from these values of p, n, D, and f, calculate the difference in effect caused by different arrangements of cranks and angles in Figs. 3, 4, 5, 6, 7, and 8.

$$H = \frac{D\,n\,p\,f}{233000} = \frac{12 \times 48.5 \times 61575 \times f}{233000} = 154 \times f.$$

Then we have the loss of effect, by multiplying the coefficient for friction by 154, which will be for Fig. 3, $154 \times 1.414 = 218$ horses. The results of all these calculations are collected and compared in the accompanying table.

Nature of crank.	Angle of engine.	Loss of effect.	Useful effect.	Friction f.	Speed of the vessel.
Single . .	90°	218	570	1.414	9.90
Opposite .	90	81	709	0.527	11.25
Single . .	120	267	523	1.732	9.37
Opposite .	120	71.5	718.5	0.464	11.30
Single . .	150	296	494	1.932	9.6
Opposite .	150	43.9	746	0.285	11.55

This shows a difference of 2 miles per hour gained by the opposite crank.

Plate I.

Fig 1.

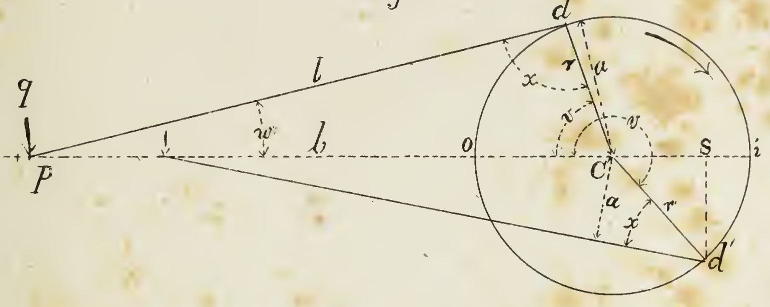
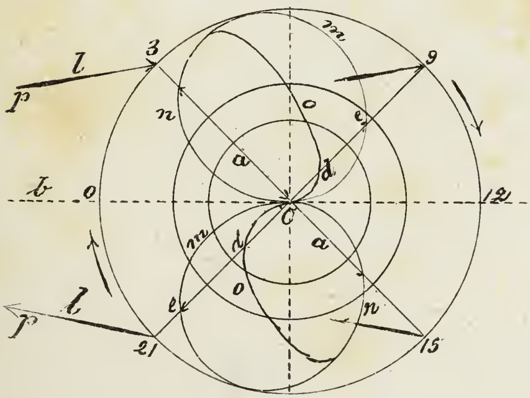


Fig 2.



J.W. Nystrom.

Plate II.

Fig. 3

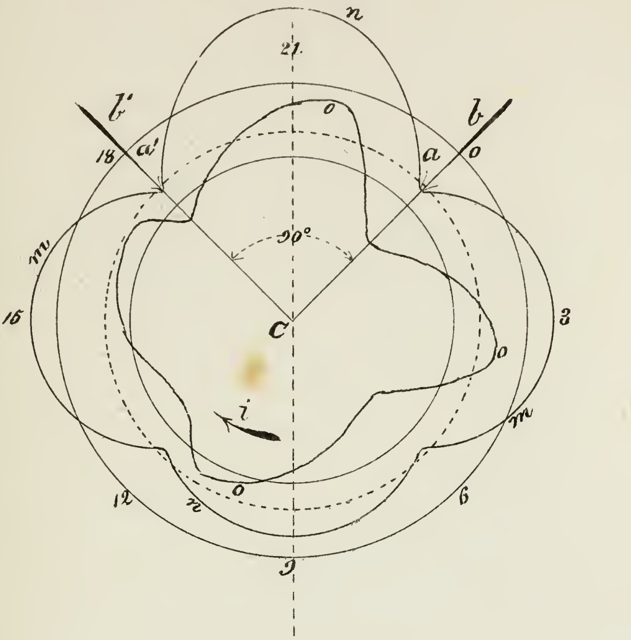
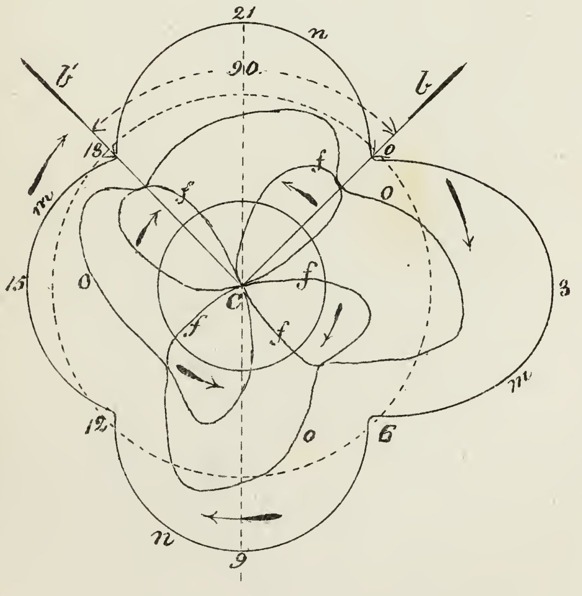


Fig 4



J.W. Nystrom

Plate III.

Fig. 5.

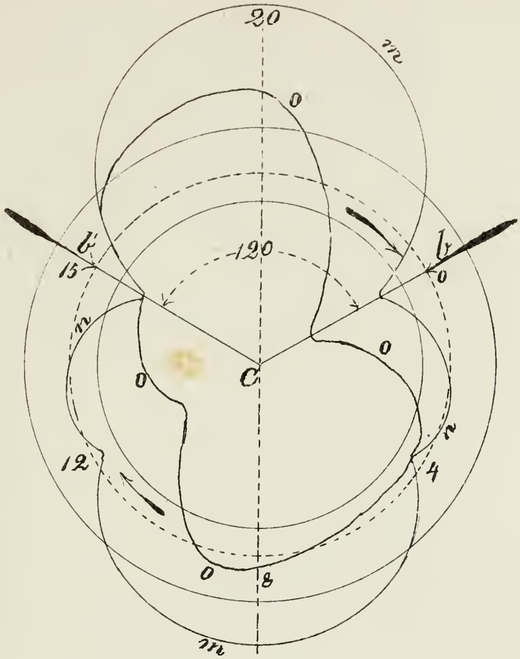
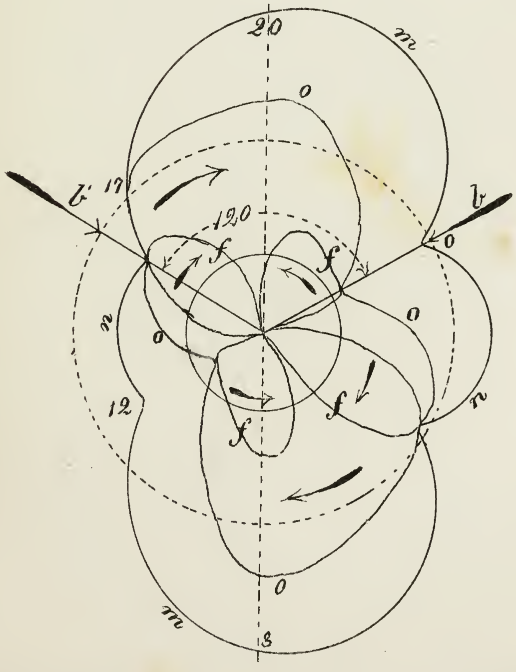


Fig. 6.



J.W.Nystrom.

Plate IV.

Fig. 7

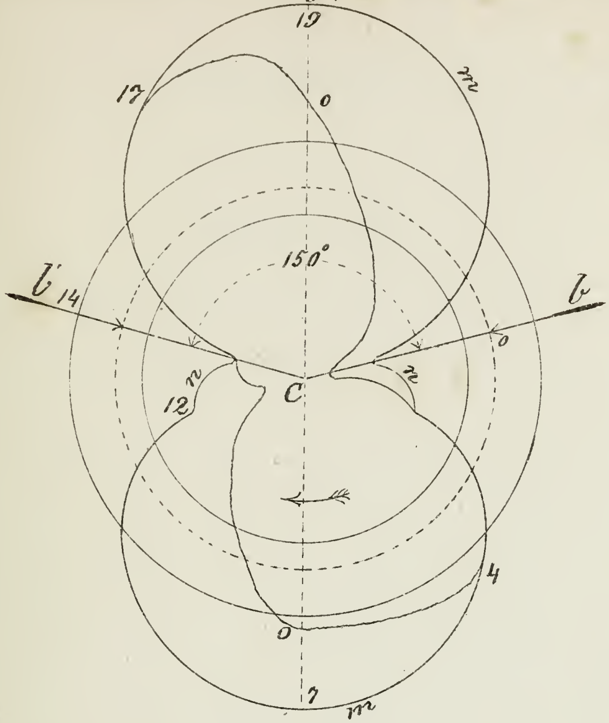
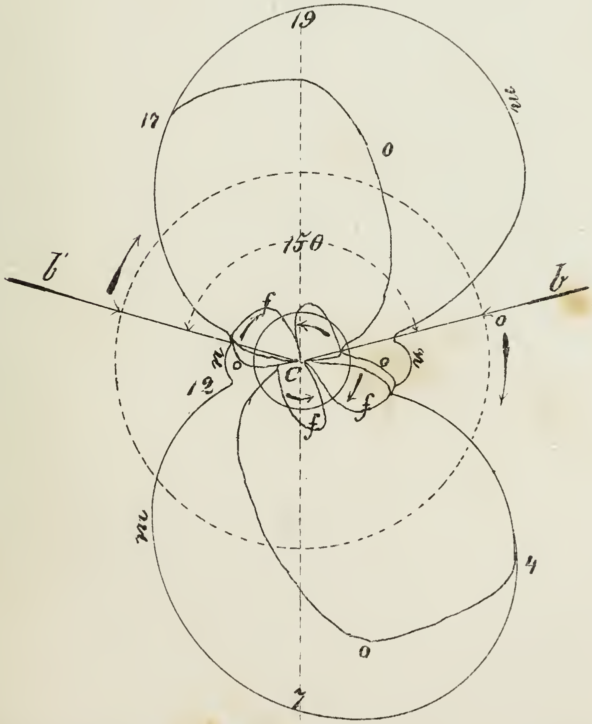


Fig. 8.



J.W. Nystrom

Tables I. & II.

One Engine working on one Crank. (Figs. 1 & 2.)

Position of crank and connecting-rod.							Steam full-stroke.		Cut off at $\frac{1}{3}$.	
n	v	w		x		b	S	a	p	ap
0	0	0°	00′	180°	00′	5.000	0.000	0.000	1.000	0.000
1	15	3	43	161	17	4.974	0.015	0.321	1.000	0.321
2	30	7	11	142	49	4.830	0.085	0.604	1.000	0.604
3	45	10	11	124	49	4.680	0.160	0.827	1.000	0.827
4	60	12	30	107	30	4.400	0.300	0.955	1.000	0.955
5	75	13	58	91	2	4.135	0.432	0.998	0.771	0.769
6	90	14	29	75	31	3.875	0.562	0.970	0.587	0.575
7	105	13	58	61	2	3.610	0.695	0.875	0.483	0.420
8	120	12	30	47	30	3.400	0.800	0.738	0.416	0.307
9	135	10	11	34	49	3.220	0.890	0.570	0.375	0.214
10	150	7	11	22	49	3.100	0.950	0.388	0.355	0.136
11	165	3	43	11	17	3.025	0.987	0.196	0.333	0.066
12	180	0	00	00	00	3.000	0.000	0.000	1.000	0.000
13	195	3	43	11	17	3.025	0.013	0.196	1.000	0.196
14	210	7	11	22	49	3.100	0.050	0.388	1.000	0.388
15	225	10	11	34	49	3.220	0.110	0.570	1.000	0.570
16	240	12	30	47	30	3.400	0.200	0.738	1.000	0.738
17	255	13	58	61	2	3.610	0.305	0.875	1.000	0.875
18	270	14	29	75	31	3.875	0.433	0.970	0.770	0.749
19	285	13	58	91	2	4.135	0.562	0.998	0.587	0.586
20	300	12	30	107	30	4.400	0.700	0.955	0.476	0.455
21	315	10	11	124	49	4.680	0.840	0.827	0.398	0.328
22	330	7	11	142	49	4.830	0.915	0.604	0.364	0.219
23	345	3	43	161	17	4.970	0.985	0.321	0.338	0.109
24	360	0	00	180	00	5.000	1.000	0.000	0.333	0.000
Mean momentum =								0.620	0.669	0.434

Table III.

Two Engines working at 90° *on one Crank.* (Fig. 3.)

		Steam full-stroke.				Cut off at $\frac{1}{3}$.			
n	v	a	a'	$a+a'$	Friction.	pa	pa'	$p(a+a')$	Friction.
0	0	0.000	0.970	0.970	Friction = 2 *sin.* $\frac{1}{2}$ 90° = 1.414.	0.000	0.575	0.575	Friction = 2 *sin.* $\frac{1}{2}$ 90° × 0.69 = 0.974.
1	15	0.321	0.875	1.196		0.321	0.420	0.741	
2	30	0.604	0.738	1.342		0.604	0.307	0.911	
3	45	0.827	0.570	1.397		0.827	0.214	1.041	
4	60	0.955	0.388	1.343		0.955	0.136	1.091	
5	75	0.998	0.196	1.194		0.769	0.066	0.835	
6	90	0.970	0.000	0.970		0.575	0.000	0.575	
7	105	0.875	0.196	1.071		0.420	0.196	0.616	
8	120	0.738	0.388	1.126		0.307	0.388	0.695	
9	135	0.570	0.570	1.140		0.214	0.570	0.784	
10	150	0.388	0.738	1.126		0.136	0.738	0.874	
11	165	0.196	0.875	1.071		0.066	0.875	0.941	
12	180	0.000	0.970	0.970		0.000	0.749	0.749	
13	195	0.196	0.998	1.194		0.196	0.526	0.722	
14	210	0.388	0.955	1.343		0.388	0.455	0.843	
15	225	0.570	0.827	1.397		0.570	0.328	0.898	
16	240	0.738	0.604	1.342		0.738	0.219	0.957	
17	255	0.875	0.321	0.196		0.875	0.109	0.984	
18	270	0.970	0.000	0.970		0.749	0.000	0.749	
19	285	0.998	0.321	1.319		0.586	0.321	0.904	
20	300	0.955	0.604	1.559		0.455	0.604	1.059	
21	315	0.827	0.827	1.654		0.328	0.827	1.155	
22	130	0.604	0.955	1.559		0.219	0.955	1.174	
23	345	0.321	0.998	1.319		0.109	0.769	0.878	
24	360	0.000	0.970	0.970		0.000	0.575	0.575	
Mean momentum = 1.240					1.414			0.868	0.974

Table IV.

Two Engines working at 90° *on opposite Cranks.* (Fig. 4.)

		Steam full-stroke.			Friction.	Cut off at $\frac{1}{3}$.			Friction.
n	v	a	a'	$a+a'$	$\pm a \mp a'$	pa	pa'	$p(a+a')$	$p(\pm a \mp a')$
0	0	0.000	0.970	0.970	0.970	0.000	0.749	0.749	0.749
1	15	0.321	0.998	1.319	0.677	0.321	0.586	0.907	0.265
2	30	0.604	0.955	1.559	0.351	0.604	0.455	1.059	0.149
3	45	0.827	0.827	1.654	0.000	0.827	0.328	1.155	0.499
4	60	0.955	0.604	1.559	0.351	0.955	0.219	1.174	0.736
5	75	0.998	0.321	1.319	0.677	0.769	0.109	0.878	0.660
6	90	0.970	0.000	0.970	0.970	0.575	0.000	0.575	0.575
7	105	0.875	0.321	1.196	0.554	0.420	0.321	0.741	0.099
8	120	0.738	0.604	1.342	0.134	0.307	0.604	0.911	0.297
9	135	0.570	0.827	1.397	0.257	0.214	0.827	1.041	0.613
10	150	0.388	0.955	1.343	0.567	0.136	0.955	1.091	0.819
11	165	0.196	0.998	1.194	0.802	0.066	0.769	0.835	0.709
12	180	0.000	0.970	0.970	0.970	0.000	0.575	0.575	0.575
13	195	0.196	0.875	1.071	0.679	0.196	0.420	0.616	0.224
14	210	0.388	0.738	1.126	0.350	0.388	0.307	0.695	0.081
15	225	0.570	0.570	1.140	0.000	0.570	0.214	0.784	0.356
16	240	0.738	0.388	1.126	0.350	0.738	0.136	0.874	0.602
17	255	0.875	0.196	1.071	0.679	0.875	0.066	0.741	0.809
18	270	0.970	0.000	1.970	0.970	0.749	0.000	0.749	0.749
19	285	0.998	0.196	1.194	0.802	0.586	0.196	0.782	0.390
20	300	0.955	0.388	1.343	0.567	0.455	0.388	0.843	0.067
21	315	0.827	0.570	1.397	0.257	0.328	0.578	0.898	0.242
22	330	0.604	0.738	1.342	0.134	0.219	0.738	0.957	0.519
23	345	0.321	0.870	1.191	0.554	0.109	0.875	0.984	0.766
24	360	0.000	0.970	0.970	0.970	0.000	0.749	0.749	0.749
Mean momentum = 1.240					0.527			0.868	0.454

Table V.

Two Engines working at 120° *on one Crank.* (Fig. 5.)

		Steam full-stroke.				Cut off at $\frac{1}{3}$.			
n	v	a	a'	$a+a'$	Friction.	pa	pa'	$p(a+a')$	Friction.
0	0	0.000	0.738	0.738	Friction $= 2\ sin.\ \frac{1}{2}\ 120° = 1.732.$	0.000	0.307	0.307	Friction $= 2\ sin.\ 120° \times 0.69 = 1.195.$
1	15	0.321	0.576	0.897		0.321	0.214	0.535	
2	30	0.604	0.388	0.992		0.604	0.136	0.740	
3	45	0.827	0.196	1.023		0.827	0.066	0.893	
4	60	0.955	0.000	0.955		0.955	0.000	0.955	
5	75	0.998	0.196	1.194		0.769	0.196	0.965	
6	90	0.970	0.388	1.358		0.575	0.388	0.963	
7	105	0.875	0.570	1.445		0.420	0.570	0.990	
8	120	0.738	0.738	1.476		0.307	0.738	1.045	
9	135	0.570	0.875	1.445		0.214	0.875	1.089	
10	150	0.388	0.970	1.358		0.136	0.749	0.885	
11	165	0.196	0.998	1.194		0.066	0.586	0.652	
12	180	0.000	0.955	0.955		0.000	0.455	0.455	
13	195	0.196	0.827	1.023		0.196	0.328	0.524	
14	210	0.388	0.604	0.992		0.388	0.219	0.607	
15	225	0.570	0.321	0.891		0.570	0.109	0.679	
16	240	0.738	0.000	0.738		0.738	0.000	0.738	
17	255	0.875	0.321	1.196		0.875	0.321	1.196	
18	270	0.970	0.604	1.574		0.749	0.604	1.353	
19	285	0.998	0.827	1.825		0.586	0.827	1.413	
20	300	0.955	0.955	1.910		0.455	0.955	1.410	
21	315	0.827	0.998	1.825		0.328	0.769	1.097	
22	330	0.604	0.970	1.574		0.979	0.575	0.794	
23	345	0.321	0.875	1.196		0.109	0.420	0.529	
24	360	0.000	0.738	0.738		0.000	0.307	0.307	
Mean momentum $= 1.240$					1.732			0.868	1.195

TABLE VI.

Two Engines working at 120° *on opposite Cranks.* (Fig. 6.)

		Steam full-stroke.			Friction.	Cut off at $\frac{1}{3}$.			Friction.
n	v	a	a'	$a+a'$	$\pm a \mp a'$	pa	pa'	$p(a+a')$	$p(\pm a \mp a')$
0	0	0.000	0.955	0.955	0.955	0.000	0.455	0.455	0.455
1	15	0.321	0.827	1.148	0.506	0.321	0.328	0.649	0.007
2	30	0.604	0.604	1.208	0.000	0.604	0.219	0.823	0.385
3	45	0.827	0.321	1.148	0.506	0.827	0.109	0.936	0.718
4	60	0.955	0.000	0.955	0.319	0.955	0.000	0.955	0.955
5	75	0.998	0.321	1.319	0.676	0.769	0.321	1.090	0.448
6	90	0.970	0.604	1.574	0.366	0.575	0.604	1.179	0.029
7	105	0.875	0.827	1.702	0.048	0.420	0.827	1.247	0.407
8	120	0.738	0.955	1.693	0.217	0.307	0.955	1.262	0.648
9	135	0.570	0.998	1.568	0.428	0.214	0.769	0.983	0.555
10	150	0.388	0.970	1.358	0.582	0.136	0.575	0.711	0.439
11	165	0.196	0.875	1.071	0.679	0.066	0.420	0.486	0.354
12	180	0.000	0.738	0.738	0.738	0.000	0.370	0.307	0.307
13	195	0.196	0.570	0.766	0.374	0.196	0.214	0.410	0.018
14	210	0.388	0.388	0.776	0.000	0.388	0.136	0.524	0.252
15	225	0.570	0.196	0.776	0.374	0.570	0.066	0.636	0.504
16	240	0.738	0.000	0.738	0.738	0.738	0.000	0.738	0.738
17	255	0.875	0.196	1.071	0.679	0.875	0.196	1.071	0.679
18	270	0.970	0.388	1.358	0.582	0.749	0.388	1.137	0.441
19	285	0.998	0.570	1.568	0.428	0.586	0.570	1.156	0.016
20	300	0.955	0.738	1.697	0.217	0.455	0.738	1.193	0.283
21	315	0.827	0.875	1.702	0.048	0.328	0.875	1.203	0.547
22	330	0.604	0.970	1.574	0.366	0.219	0.749	0.968	0.530
23	345	0.321	0.998	1.319	0.677	0.109	0.586	0.695	0.477
24	360	0.000	0.955	0.955	0.955	0.000	0.455	0.455	0.455
Mean momentum = 1.210					0.464			0.868	0.414

Table VII.

Two Engines working at 150° *on one Crank.* (Fig. 7.)

		Steam full-stroke.				Cut off at $\frac{1}{3}$.			
n	v	a	a'	$a+a'$	Friction.	pa	pa'	$p\ (a+a)$	Friction.
0	0	0.000	0.388	0.388	Friction $= 2\ sin.\ \frac{1}{2}\ 150° = 1.9318.$	0.000	0.136	0.136	Friction $= 2\ sin.\ \frac{1}{2}\ 150° \times 0.69 = 1.333$
1	15	0.321	0.196	0.517		0.321	0.066	0.387	
2	30	0.604	0.000	0.604		0.604	0.000	0.604	
3	45	0.827	0.196	1.023		0.827	0.196	1.023	
4	60	0.955	0.388	1.343		0.955	0.388	1.343	
5	75	0.998	0.570	1.563		0.769	0.570	1.339	
6	90	0.970	0.738	1.708		0.575	0.738	1.313	
7	105	0.875	0.875	1.750		0.420	0.875	1.295	
8	120	0.738	0.970	1.708		0.307	0.749	1.056	
9	135	0.570	0.998	1.568		0.214	0.586	0.800	
10	150	0.388	0.955	1.343		0.136	0.455	0.591	
11	165	0.196	0.827	1.023		0.066	0.328	0.394	
12	180	0.000	0.604	0.604		0.000	0.219	0.219	
13	195	0.196	0.321	0.517		0.196	0.109	0.305	
14	210	0.388	0.000	0.388		0.388	0.000	0.388	
15	225	0.570	0.321	0.891		0.570	0.321	0.891	
16	240	0.738	0.604	1.342		0.738	0.604	1.342	
17	255	0.875	0.827	1.702		0.875	0.827	1.702	
18	270	0.970	0.955	1.925		0.749	0.955	1.704	
19	285	0.998	0.998	1.996		0.586	0.769	1.355	
20	300	0.955	0.970	1.925		0.455	0.575	1.030	
21	315	0.827	0.875	1.702		0.328	0.420	0.748	
22	330	0.604	0.738	1.342		0.219	0.307	0.526	
23	345	0.321	0.570	0.891		0.109	0.214	0.323	
24	360	0.000	0.388	0.388		0.000	0.136	0.136	
Mean momentum $= 1.240$					1.9318			0.868	1.333

Table VIII.

Two Engines working at 150° *on opposite Cranks.* (Fig. 8.)

		Steam full-stroke.			Friction.	Cut off at $\frac{1}{3}$.			Friction.
n	v	a	a'	$a+a'$	$\pm a \mp a'$	pa	pa'	$p(a+a')$	$p(\pm a \mp a')$
0	0	0.000	0.604	0.604	0.604	0.000	0.219	0.219	0.219
1	15	0.321	0.321	0.642	0.000	0.321	0.109	0.430	0.212
2	30	0.604	0.000	0.604	0.604	0.604	0.000	0.604	0.604
3	45	0.827	0.321	1.148	0.506	0.827	0.321	1.148	0.506
4	60	0.955	0.604	1.559	0.351	0.955	0.604	1.559	0.351
5	75	0.998	0.827	1.825	0.171	0.769	0.827	1.596	0.058
6	90	0.970	0.955	1.925	0.025	0.575	0.955	1.530	0.380
7	105	0.875	0.998	1.873	0.123	0.420	0.769	1.189	0.349
8	120	0.738	0.970	1.708	0.232	0.307	0.575	0.882	0.268
9	135	0.570	0.875	1.445	0.305	0.214	0.420	0.634	0.206
10	150	0.388	0.738	1.126	0.350	0.136	0.307	0.443	0.171
11	165	0.196	0.570	0.766	0.374	0.066	0.214	0.280	0.148
12	180	0.000	0.388	0.388	0.388	0.000	0.136	0.136	0.136
13	195	0.196	0.196	0.392	0.000	0.196	0.066	0.262	0.130
14	210	0.388	0.000	0.388	0.388	0.388	0.000	0.388	0.388
15	225	0.570	0.196	0.766	0.374	0.570	0.196	0.766	0.374
16	240	0.738	0.388	1.126	0.350	0.738	0.388	1.126	0.350
17	255	0.875	0.570	1.445	0.305	0.875	0.570	1.445	0.305
18	270	0.970	0.738	1.708	0.232	0.749	0.738	1.487	0.011
19	285	0.998	0.875	1.873	0.123	0.586	0.875	1.461	0.289
20	300	0.955	0.970	1.925	0.015	0.455	0.749	1.204	0.294
21	315	0.827	0.998	1.825	0.171	0.328	0.586	0.914	0.258
22	330	0.604	0.955	1.549	0.351	0.219	0.455	0.674	0.236
23	345	0.321	0.827	1.148	0.506	0.109	0.328	0.437	0.219
24	360	0.000	0.604	0.604	0.604	0.000	0.219	0.219	0.219
Mean momentum = 1.240					0.285			0.868	0.265

LENGTH OF THE CONNECTING-ROD.

The length of the connecting-rod in a propeller-engine deserves some attention. It is evident that the longer it is, the better; but the question is, how will it affect the quality of the steam-engine, and economy of power? In reference to the arrangement of the steam-engine, a short connecting-rod has the advantage of occupying less room, and thereby allowing a narrow space for the steam-engine; the disadvantage is, it gives a more irregular motion to the machinery, which two points are of some consequence when the engine is direct action, and works fast. To the economy of power, we make a calculation for four different lengths, 1, 2, and 3 times the stroke, and one infinite. It is evident that the loss of effect consists, first, in the mean momentum for rotation; second, the additional friction in the guides. The former will be calculated, as before described, by the formulæ.

$$sin.w = \frac{r\ sin.v}{l},$$

$$a = \frac{l\ sin.w\ sin.x}{sin.v},$$

$$s = \frac{l + r}{2} - \frac{l\ sin.x}{2\ sin.v},$$

In the accompanying Tables, IX., X., and XI., are the values calculated for three different connecting-rods, with their frictions in the guides. When the connecting-rod is infinite, the momentum $a = sin.v$ and the friction in the guides $= o$, because the angle $w = o$. When the connecting-rod is short, in every position of the crosshead in the guide, will be a friction $= tang.w'$ calculated and collected, as seen in the tables. $s =$ the space which the piston moves between each 24th division.

$q = p \; tang.w$ pressure in the guides.
$p =$ total pressure on the piston.
$q \, s =$ friction in each division.

The sum of those frictions, multiplied by $0.06 =$ coefficient for frictions in guides, will be the true friction in a fraction, of the total effect.

In the column where the connecting-rod $l = \infty$ infinite, the mean momentum for rotation is 0.6325 (proper number = 0,63694). When the connecting-rod is equal to the stroke, the mean momentum = 0.577.

$z =$ momentum from the short connecting-rod compared with the infinite one.

$$0.6325 : 0.577 = 1 : {}^1z$$

connecting-rods of

$$\left.\begin{array}{lll}
\text{equal the stroke} & {}^1z = \dfrac{0.577}{0.6325} & = 0.911 \\[2ex]
\text{twice} \quad \text{``} & {}^2z = \dfrac{0.620}{0.6325} & = 0.981 \\[2ex]
\text{three times} \quad \text{``} & {}^3z = \dfrac{0.626}{0.6325} & = 0.990 \\[2ex]
\text{infinite} \quad \text{``} & {}^\infty z = \dfrac{0.6325}{0.6325} & = 1.00
\end{array}\right\} \begin{array}{c}\text{useful effect} \\ \text{omitting} \\ \text{friction.}\end{array}$$

Now we will collect the results from the Tables IX., X., and XI., to Table XII., in which we will find that the useful effect by an infinite connecting-rod, is to the useful effect of a short one, nearly as 1 is to *cosin.* for the greatest angle of the connecting-rod to the centre line of the engine.

$$\text{Useful effect} = \frac{\sqrt{l^2 - r^2}}{l}.$$

We see here that the difference between useful effect by connecting-rods 2 and 3 times the stroke, is only 0.0179; consequently, when the connecting-rod can be twice the stroke, there will be no remuneration in the effort to make it longer if the space is narrow to place it in.

To thoroughly test the above treatment on screw-propellers and their steam-engines, it would require three different vessels; one very sharp, and one with common proportion, and the third one very full. If circumstances would allow the vessels to have about the same draught of water, so that the same diameter of propellers could be used on them, it would save number of propellers which would require at least six different kinds. Experiments on smaller boats could be easily tried. From such experiments should be obtained results accompanied with rules, so that for any disruption of vessels, or arrangement of engines, the rules would give a corresponding propeller.

After the experiments, the propellers need not be thrown away as useless; they would all be good for their corresponding arrangement. Money or time should not be spent in making the propeller-blades of any peculiar shape on their extremities or edges, like a tail of a fish, &c.

The steam-engines would suffice with the three, one in each vessel. The two with direct-action steam-engines, and the other with gearing, and, if convenient, to change the gearing one to different gears.

Table IX.

Connecting-rod equal to the Stroke.

Position of crank and connecting-rod.						Friction in the guide.		
n	v	w		a	S	s	$tang.w = q$	qs
0	00	0°	00′	0.000	0.000	0.000	0.000	0.00000
1	15	7	25	0.380	0.026	0.026	0.130	0.00338
2	30	14	30	0.702	0.095	0.069	0.259	0.01790
3	45	20	40	0.925	0.188	0.093	0.377	0.03512
4	60	25	40	0.998	0.345	0.157	0.480	0.07550
5	75	28	50	0.974	0.489	0.144	0.550	0.07940
6	90	30	00	0.868	0.632	0.133	0.577	0.07700
7	105	28	50	0.718	0.753	0.121	0.550	0.06670
8	120	25	40	0.562	0.850	0.097	0.480	0.04660
9	135	20	40	0.413	0.913	0.063	0.377	0.02380
10	150	14	30	0.267	0.966	0.053	0.259	0.01375
11	165	7	25	0.149	0.995	0.029	0.130	0.00378
12	180	0	00	0.000	0.000	0.005	0.000	0.00000
13	195	7	25	0.149	0.005	0.005	0.130	0.00065
14	210	14	30	0.267	0.034	0.029	0.259	0.00752
15	225	20	40	0.413	0.087	0.053	0.377	0.02000
16	240	25	40	0.562	0.150	0.063	0.480	0.03030
17	255	28	50	0.718	0.247	0.097	0.550	0.05345
18	270	30	00	0.868	0.368	0.121	0.577	0.07000
19	285	28	50	0.974	0.511	0.143	0.550	0.07885
20	300	25	40	0.998	0.655	0.144	0.488	0.07040
21	315	20	40	0.925	0.818	0.157	0.377	0.05930
22	330	14	30	0.702	0.905	0.093	0.259	0.02411
23	345	7	25	0.380	0.974	0.069	0.130	0.00898
24	360	0	00	0.000	0.000	0.026	0.000	0.00000
Mean momentum =				0.577				0.86649

Table X.

Connecting-rod twice the stroke.

Position of crank & connecting-rod.			Steam full-stroke.		Friction in the guide.			$l = \infty$
n	v	w	a	S	s	$tang. w = q$	qs	$sin. v = a$
0	0	0° 00′	0.000	0.000	0.000	0.0000	0.0000	0.000
1	15	3 43	0.321	0.015	0.015	0.0649	0.000976	0.259
2	30	7 11	0.604	0.085	0.070	0.1260	0.0084	0.500
3	45	10 11	0.827	0.160	0.075	0.1796	0.0135	0.707
4	60	12 30	0.955	0.300	0.140	0.2216	0.0311	0.866
5	75	13 58	0.998	0.432	0.132	0.2487	0.0329	0.966
6	90	14 29	0.970	0.562	0.130	0.2583	0.0350	1.000
7	105	13 58	0.875	0.695	0.128	0.2487	0.03185	0.966
8	120	12 30	0.738	0.800	0.105	0.2216	0.0235	0.866
9	135	10 11	0.570	0.890	0.090	0.1796	0.0162	0.707
10	150	7 11	0.388	0.950	0.060	0.1260	0.00757	0.500
11	165	3 43	0.196	0.987	0.037	0.0649	0.00241	0.259
12	180	0 00	0.000	0.000	0.013	0.0000	0.0000	0.000
13	195	3 43	0.196	0.013	0.013	0.0649	0.000845	0.259
14	210	7 11	0.388	0.050	0.037	0.1260	0.00467	0.500
15	225	10 11	0.570	0.110	0.060	0.1796	0.00666	0.707
16	240	12 30	0.738	0.200	0.090	0.2216	0.0200	0.866
17	255	13 58	0.875	0.305	0.105	0.2487	0.02617	0.966
18	270	14 29	0.970	0.433	0.128	0.2583	0.0331	1.000
19	285	13 58	0.998	0.562	0.130	0.2487	0.0336	0.966
20	300	12 30	0.955	0.700	0.132	0.2216	0.0293	0.866
21	315	10 11	0.827	0.840	0.140	0.1796	0.0251	0.707
22	330	7 11	0.604	0.915	0.075	0.1260	0.00947	0.500
23	345	3 43	0.321	0.985	0.070	0.0649	0.00456	0.259
24	360	0 00	0.000	1.000	0.015	0.0000	0.0000	0.000
Mean momentum $= 0.620$							0.397681	0.6325

Table XI.

Connecting-rod 3 times the Stroke.

Position of the crank.						Friction in the guide.		
n	v	w		a	S	s	$tang.w = q$	qs
0	00	0°	00′	0.000	0.000	0.000	0.000	0.00000
1	15	2	28	0.300	0.020	0.020	0.043	0.00186
2	30	4	47	0.571	0.070	0.050	0.083	0.00416
3	45	6	45	0.784	0.160	0.090	0.118	0.01065
4	60	8	17	0.927	0.270	0.110	0.145	0.01600
5	75	9	15	0.996	0.400	0.130	0.163	0.02122
6	90	9	35	0.985	0.543	0.143	0.169	0.02422
7	105	9	15	0.911	0.666	0.123	0.163	0.02012
8	120	8	17	0.784	0.777	0.111	0.145	0.01610
9	135	6	45	0.618	0.866	0.089	0.118	0.01050
10	150	4	47	0.426	0.934	0.068	0.083	0.00564
11	165	2	28	0.217	0.974	0.040	0.043	0.00172
12	180	0	00	0.000	1.000	0.026	0.000	0.00000
13	195	2	28	0.217	0.026	0.026	0.043	0.00112
14	210	4	47	0.426	0.066	0.040	0.083	0.00332
15	225	6	45	0.168	0.134	0.068	0.118	0.00803
16	245	8	17	0.784	0.223	0.089	0.145	0.01293
17	255	9	15	0.911	0.334	0.111	0.163	0.01814
18	270	9	35	0.985	0.457	0.123	0.169	0.02083
19	285	9	15	0.996	0.600	0.143	0.163	0.02334
20	300	8	17	0.927	0.730	0.130	0.145	0.01887
21	315	6	45	0.784	0.860	0.110	0.118	0.01300
22	330	4	47	0.571	0.930	0.090	0.083	0.00746
23	345	2	28	0.300	0.980	0.050	0.043	0.00215
24	360	0	00	0.000	1.000	0.020	0.000	0.00000
Mean momentum =				0.626				0.24887

Table XII.

	Connecting-rod times the stroke.			
	1	2	3	∞
Mean momentum a . .	0.5770	0.6200	0.6260	0.6325
Friction qs	0.8665	0.3977	0.2488	0.0000
Friction 0.06 qs . . .	0.0520	0.0238	0.0149	0.0000
Momentum z	0.9110	0.9810	0.9900	1.0000
Useful effect z—0.06 qs . .	0.8590	0.9572	0.9751	1.0000
When $v = 90°$, $cos.w =$.	0.8660	0.9682	0.9860	1.0000

DESCRIPTION

OF A

CENTRIPETAL SCREW-PROPELLER.

PATENTED BY J. W. NYSTROM.

PHILADELPHIA, MARCH 19, 1850.

WHEREAS in screw-propellers the water between the blades is acted upon at the same moment by two forces, the one being the propulsive force, resulting from the oblique action of the revolving blades, and the other being the centrifugal force generated by their rotation; the first force tending to force the water backwards, in direction parallel to the axis of the propeller, and the second force, tending to force it outwards, in direction at right angle to the axis or centre line. Now, my invention consists in counteracting the centrifugal force by a particular curve imparted to the blades of the propeller in such a manner that the water, instead of being deflected outwards, is delivered in direction parallel to the axis of the propeller.

The formulæ by which the curvature of the blades is calculated, are deduced from those by which the value of the centrifugal force is obtained, in the following manner:—

Let a body B (**Fig. 1,** Plate V.) revolve (in the direction of the arrow) round a point A, at a distance of r feet from that central point, with a velocity of h feet per second; its centrifugal force will then be given by the equation.

$$\text{Centrifugal force } C = \frac{B\,h^2}{g\,r} \quad . \quad . \quad . \quad (1)$$

In which equation B denotes the weight of the body, and $g = 32.2$, the force of gravity. If the number of revolutions per minute, which may be denoted by the letter n, and the radius r, be known, the velocity per second, or the quantity h in the equation 1, is found by the formula

$$h = \frac{2\,\pi\,r\,n}{60} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By substituting this value of h in the equation 1, we obtain—

$$C = B\,\frac{4\,r^2\,\pi^2\,n^2}{60^2\,r\,g} = \mathrm{B}\,\frac{4\,r\,\pi^2\,n^2}{60^2\,g} \quad . \quad . \quad (3)$$

the value of the centrifugal force in terms of the quantities, B, r, π, n, and g.

Now, referring again to **Fig. 1,** let the line $a\,b$ represent the direction and momentum of the body B, under the action of the propulsive force, and $a\,c$ the magnitude and direction of a force equal and opposite to the centrifugal force acting upon it; then, by a well-known principle of mechanics, the diagonal $a\,d$ of the parallelogram erected on the two forces as sides, will represent the magnitude and direction of their resultant force, which will force the body to describe a circle around the centre A, with a radius of r feet, and with a velocity of h feet per second, if then the blades of the propeller at this distance of r feet from the centre line be at right angles to this diagonal line, it will counteract the tendency of the water, which is the body in this instance, to pass outwards from the axis under the action of the centrifugal force.

The angle v, which the blades at this distance from the axis make with the radius, and which is equal to the angle $d\ a\ b$, which the diagonal of the parallelogram makes with the line $a\ b$, is determined by the trigonometrical equation

$$tang.v = \frac{sin.v}{cos.v}.$$

In the parallelogram $a\ b\ c\ d$, we have

$$a\ c = sin.v = C,$$

and

$$a\ b = cos.v = B,$$

hence

$$tang.v = \frac{C}{B},$$

and

$$C = B\ tang.v.$$

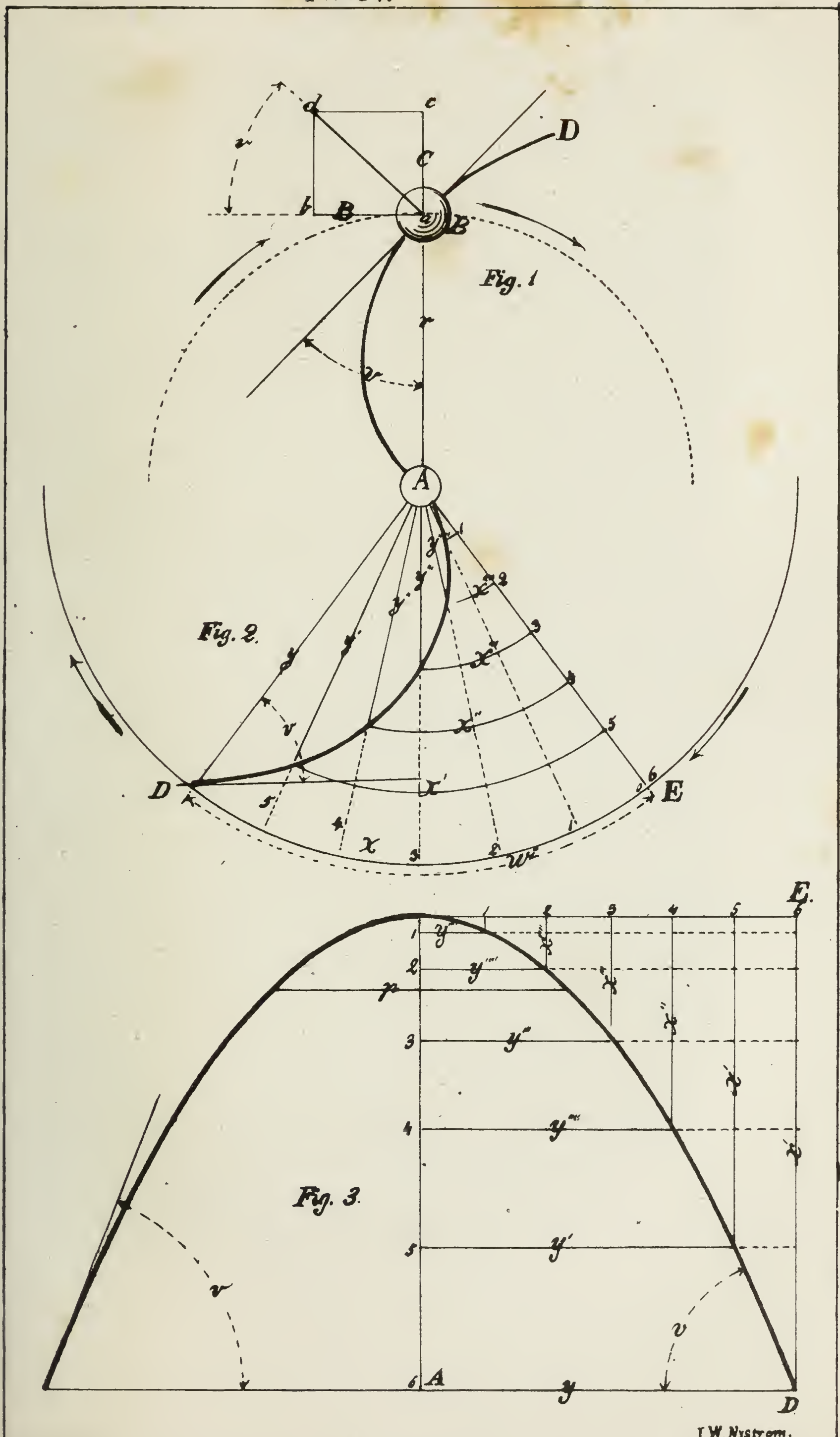
Substituting in this equation the value of C, given by equation 3, we have

$$C = B \frac{4\ r\ \pi^2\ n^2}{60^2\ g} = B\ tang.v,$$

of which

$$tang.v = \frac{4\ r\ \pi^2\ n^2}{60^2\ g}.$$

Plate V.



J. W. Nystrom.

By replacing the symbols π and g by their known value, viz. $g = 32.2$, $\pi = 3.1416$, we have

$$tang. v = \frac{4\ r\ n^2}{11740} = \frac{D\,n^2}{5870}, \quad . \quad . \quad . \quad . \qquad (4)$$

which equation represents a rule for finding the angle v, which may be thus expressed in words: Multiply the number of revolutions of the propeller per minute by itself, and the product by the diameter D of the circle described, in feet; divide the product by 5870; the quotient will be the tangent of the angle which the blades of the propeller should make with the radius at the circumference of the circle described.

If the inclination of the blades at various distances from the axis be determined by this rule, and if the inclined lines be drawn and united, a spiral curve, $A\ D$ (**Figs. 1** and **2**, Plate V.), will be formed, progressively increasing in its inclination to the radii of the propeller as it proceeds from the centre to the circumference, and this curve will be contained in the sector of the circle bounded by two radii, $A\ D$ and $A\ E$, and the included arc $D\ E$ or $w°$. In order to save the time and figuring required to calculate the inclination of the propeller-blades at all the intermediate points between the axis A and the circumference B, we will have recourse to some formulæ of a common arithmetical spiral. (See Fig. 2.) The tangent for the angle v bears the following analogy to the radius y and the circle-arc x that

$$tang. v : 1 = x : y,$$

and

$$tang.v = \frac{x}{y}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By reference to the formulæ for the circle-arc x we have

$$x = \frac{\pi\ r\ w^\circ}{180}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which $r = y$, the radii of which the angle v is calculated; $w^\circ =$ the angle in degrees in which the circle-arc x and the spiral $A\ D$ are contained. By insertion of this value of x in the formulæ, we have

$$tang.v = \frac{\pi\ r\ w^\circ}{180\ y} = \frac{\pi\ w^\circ}{180}.$$

As the *tang.v* is given by the formula 4, we have

$$tang.v = \frac{D\,n^2}{5870} = \frac{\pi\ w}{180}, \quad . \quad . \quad . \quad . \quad (7)$$

from which we obtain the angle

$$w = \frac{180\ D\ n^2}{5870\ \pi} = \frac{D\ n^2}{102.4}, \quad . \quad . \quad . \quad . \quad (8)$$

This is the formula by which the curvature of the spiral is ascertained, but, before applying it to the propeller, it must undergo a little modification in reference to the quantity n, depending on the slip of the propeller.

It is evident that, if the propeller has no slip, it does not act to propel the vessel, as the water is a free movable body, and can give no more resistance than that of its own inertia, and the hydrostatic pressure of water and air. If there is no slip, the resistance by the inertia will be o, and the hydrostatic pressure of the water and air will be equal on both sides of the propeller; therefore, *slip is a necessity for propulsion in water*, and, accordingly, a measure of propulsion. If a propeller makes n revolutions per minute, and has a slip $= S$, the value of propulsion is measured by the product of $n\,S$; that if a propeller makes $n = 50$ revolutions per minute, and has a slip $S = \frac{1}{2}$, the number of revolutions which act to propel will be only $n\,S = 50 \times \frac{1}{2} = 25$ revolutions; therefore, the value n in the formula 7 must be multiplied by the slip $= S$, when n denotes the total number of revolutions per minute, and

$$w^\circ = \frac{D\,n^2\,S^2}{102.4}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This is the practical rule for calculating the angle w°, expressed in words:—

Rule.—Multiply the diameter of the propeller by the square of the number of revolutions per minute, and by the square of the slip in a fraction; divide the last product by 102.4; the quotient will be the angle w° expressed in degrees of the circle.

When this angle w° is obtained, set it at the circumference of the propeller, then, as the curve is a regular arithmetical spiral, it is described by dividing the angle w° into a number of equal parts, 6 for example, as seen in **Fig. 2,** and number them in regular succession from one extremity D of the arc to the other E; then divide the radii $A\ E$ of the propeller into the same number of equal parts (6), and number them in regular succession from the circumference E to the centre A; through the divisions 1, 2, 3, &c., of the angle $w°$, draw the radii y', y'', y''', &c., and through the divisions of the radii $E\ A$, with the axis A of the propeller as a centre, draw the circular arcs x', x'', x''', &c.; unite the points of insertion of the arcs x', x'', x''', &c., with their respective radii y', y'', y''', &c., by a curved line, which will be the spiral curve required in the propeller-blades, moving in the direction indicated by the arrow, with n revolutions per minute, to counterbalance the tendency of the water to move outwards from the axis of the propeller under the influence of the centrifugal force, generated by the rotation.

This curve is not influenced by the obliquity of the propeller-blades to its axis, or by what is generally termed its *pitch*, which may be adjusted at pleasure to suit the engine and vessel. As the water is discharged by the propeller in a direction parallel with its axis, the slip of the propeller will be less than with those constructed by the ordinary, or with straight blades.

The centripetal propeller is a *regular screw*. Its peculiarity from others consists only in that the *generatrix* for the screw is a curved line (spiral) drawn on a plane at right angle to the axis of the screw, while a common screw-propeller has a straight *generatrix* drawn on the plane at right angle to the axis.

For canal propeller-boats, where it is of great importance to prevent, as far as possible, the propeller from agitating the water, that is wholly overcome by a propeller constructed upon this principle, because it only touches the water within the cylinder described by the propeller, and delivers the water as solid as it receives it; which is not the case with the straight-bladed propeller, where the water is thrown out by the centrifugal force, and thereby turbids the same, and spoils the bottom and sides of the canal, and causes a partial vacuum behind the propeller, and the rudder comes in a porous mass of water; which cannot be the case when the propeller is centripetal, and then the centrifugal force acts to propel the vessel, and the rudder comes in a solid mass of water, while an easier steering of the vessel is obtained.

The accompanying drawing, on Plates VII., **Figs. 4** and **5**, represents a propeller made of wrought-iron. The blades are screwed together at the hub, so that any of the blades can be taken off when required. The hub can be made of cast or wrought-iron, as a round cylinder, to which the blades are screwed. The blades are supported by two bands at the circumference, so that, if the propeller meets with any obstacle in the water, the bands serve to keep the blades in their proper position, and, as the blades are curved, the resistance and centrifugal force of the water act to strengthen the blades, then, if there were no bands around them, the propeller would obtain a larger diameter which might not be allowed in its determined space. Therefore, a pair of narrow bands around the propeller, particularly when made of wrought-iron, is necessary. If made of cast-iron, the blades would sooner break than bend. We will here give a few examples how to calculate the angles w° and v.

Example 1.—Suppose a canal-boat having a propeller 6 feet in diameter, and making 65 revolutions per minute, with a slip of 55 per cent., we have from the formula 9

$$w^\circ = \frac{6 \times 65^2 \times 0.55}{102.4} = 75^\circ. = 75^\circ;$$

from the formula 7 we have

$$tang.v = \frac{3.14 \times 75^\circ.}{180} = 1.31. \quad v = 53^\circ.$$

Example 2.—A merchant steam-ship having a propeller 12 feet in diameter, making 54 revolutions per minute, and so proportioned to the vessel that the slip will be 30: What will be the angles w° and v?

$$w = \frac{12 \times 54^2 \times 0.30^2}{102.4} = 30.66 = 30^\circ\ 40',$$

$$tang.v = \frac{3.14 \times 30.66}{180} = 0.535. \quad v = 28^\circ.$$

If the same propeller makes 64 revolutions per minute, and had a slip of say 38 per cent., the angle w° will then be

$$w^\circ = \frac{12 \times 64^2 \times 0.38^2}{102.4} = 69.5,$$

$$tang.v = \frac{3.14 \times 69.5}{180} = 1.2, \quad v = 50^\circ.$$

We see by this that the angles w° and v increase as the squares of the number of revolutions and slip. It is, therefore, hard, or, more correctly, impossible, to make the spiral to correspond with the number of revolutions and slip in practice. Then the question arises: how will it affect the quality of the propeller and economy of power, when the generatrix is more or less curved than given by the preceding formula? In answer to that question, it will be necessary to find out if there exists any gain of effect by counteracting the centrifugal force, and if that gain depends on the curve being particularly proportioned by the preceding formulæ.

Plate VI. is intended to display the loss of effect by the straight-bladed propeller, and gain of effect when centripetal.

When the propeller is revolving, the water between the blades is acted upon at the same moment by two forces, the one being the propulsive force resulting from the oblique action of the revolving blades, and the other being the centrifugal force generated by their rotation.

Straight-bladed Propeller.

Let the first-mentioned force be represented by the line $b\,c$, and the second one by the line $a\,c$. The line $a\,b$ will represent the motion of the water; the square of the line $a\,b$ will represent the total effect delivered from the steam-engine by the propeller; the square of the line $b\,c$ is the useful effect which is acting to propel the vessel; and the square of the line $a\,c$ the useless (loss of) effect caused by the centrifugal force.

$$\text{Total effect } (a\,b)^2 = (b\,c)^2 + (a\,c)^2$$
$$\text{Useful effect } (b\,c)^2 = (a\,b)^2 - (a\,c)^2$$
$$\text{Loss of effect } (a\,c)^2 = (a\,b)^2 - (b\,c)^2.$$

Centripetal Propeller.

Let a plane cut the propeller through its centre line; the section of the blades will be a straight or curved line, just as the pitch of the propeller is uniform or expanding (see pages 16 and 98 about expanding pitch), but whatever the nature of pitch may be, the section of the blades will have an inclination to the centre line of the propeller of an angle ϕ, so that

$$tang.\phi = \frac{180\,D}{P w^\circ} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which P denotes the pitch in the point where the inclination is to be calculated.

Plate VI.

NYSTROM'S CENTRIPETAL SCREW PROPELLER.

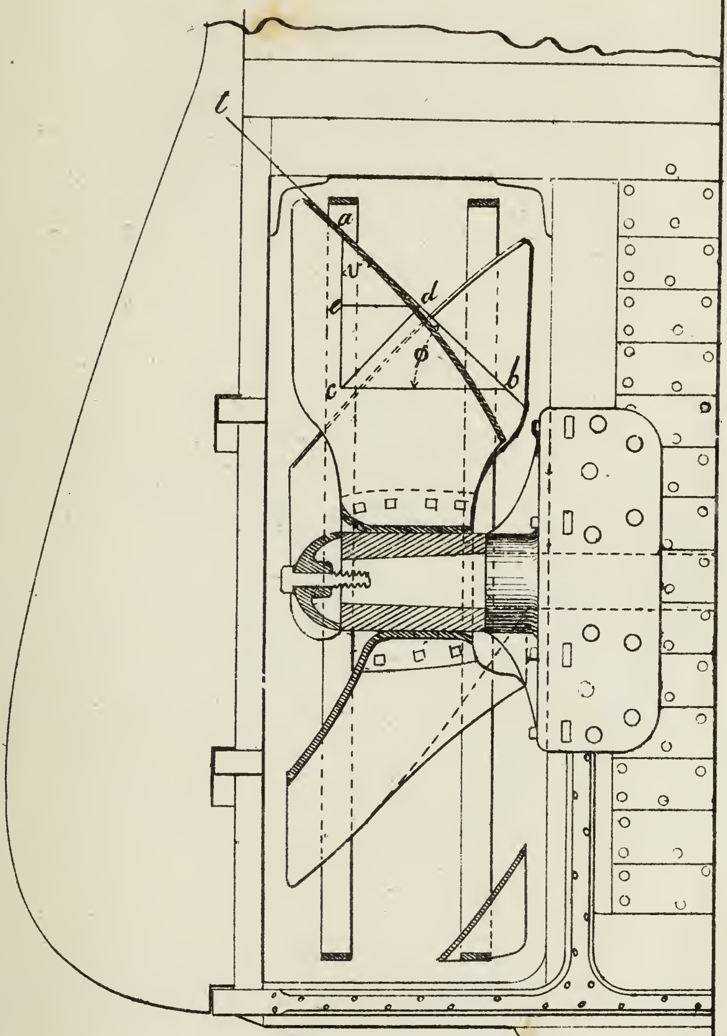


Plate. VII.

NYSTROM'S PROPELLER.

Fig. 4

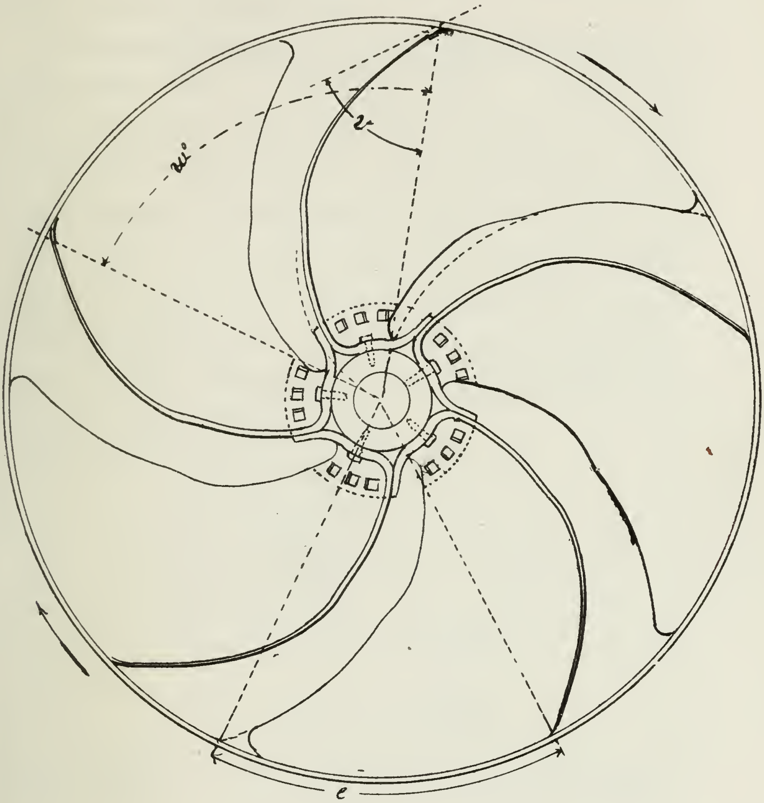
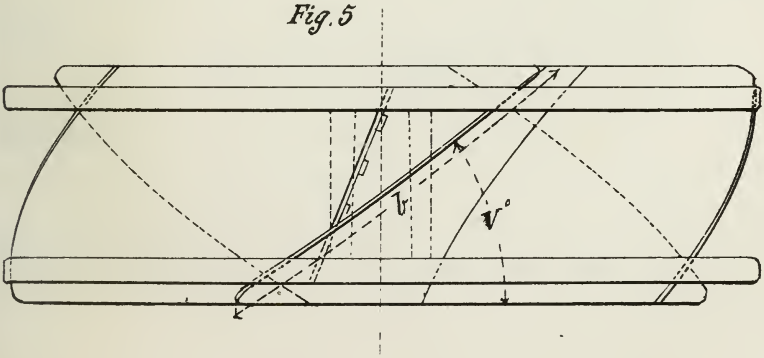


Fig. 5



J.W. Nystrom.

If the blades are straight, the section will be a straight line at right angle to the axis of the propeller or in direction of the radii.

The accompanying diagram represents the section of a propeller with an expanding pitch. Let the line $a\ c$ represent the magnitude and direction of the centrifugal force in the point a, draw through the point a the tangent $t\ b$, draw from the point c a line $c\ d$, at right angle to $a\ b$, from d draw the line de, at right angle to $a\ c$, then the line $d\ e$ represents the magnitude and direction of a force, which acts to propel the vessel in the direction parallel to its motion, and caused by the centrifugal force.

From this diagram we obtain

$$bc = ab \cos.\phi$$

$$ac = ab \sin.\phi$$

$$cd = bc \sin.\phi$$

$$de = cd \sin.\phi = bc \sin.^2\phi$$

$$bc = \frac{de}{\sin.^2\phi} = \frac{ac}{tang.\phi}$$

$$de = \frac{ac \sin.^2\phi}{tang.\phi} = \frac{ac \sin^2.\phi \cos.\phi}{\sin.\phi} = ac \sin.\phi \cos.\phi$$

but,

$$\sin.\phi \cos.\phi = \frac{\sin.}{2} 2\phi$$

therefore,

$$de = ac \frac{\sin.}{2} 2\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

That is to say, the force de is greatest when half the sine for double the inclination is greatest. The greatest sine is equal to 1, which answers to the angle 90°, therefore *de* will be greatest when $2\phi = 90°$, or

$$\phi = \frac{90}{2} = 45°.$$

That is to say, the centrifugal force acts to propel the vessel with its greatest tendency when the inclination of the section is 45° independent of slip or velocity. The angle $w°$ will then be simply

$$w° = \frac{180\ D}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The mechanical effect developed by *de* will be, when $ac = C$.

$$E = C\, P\, n\, S\, \frac{sin.}{2}\, 2\,\phi \quad . \quad . \quad . \quad . \quad . \quad (12)$$

I do *not* mean to say, that this mechanical effect E is created from nothing by the propeller and the centrifugal force itself; but, as the *steam* delivers a certain amount of effect through the engine and propeller. It is my intention to construct an instrument that will employ all this effect in the propelling of the vessel.

Before closing this book, we will construct a propeller upon this principle, and with an expanding pitch in two directions.

A Table of Formulæ collected for convenience and reference.

$$C = \frac{B\,h^2}{g\,r} \qquad (1)$$

$$h = \frac{2\,\pi\,r\,n}{60} \qquad (2)$$

$$C = \frac{B\,4\,r\,\pi^2\,n^2}{60^2\,g} \qquad (3)$$

$$tang.\,v = \frac{D\,n^2}{5870} \qquad (4)$$

$$x = \frac{\pi\,r\,w^\circ}{180} \qquad (5)$$

$$tang.\,v = \frac{\pi\,w^\circ}{180} \qquad (6)$$

$$w = \frac{D\,n^2}{102.4} \qquad (7)$$

$$w = \frac{D\,n^2\,S^2}{102.4} \qquad (8)$$

$$tang.\,\phi = \frac{180\,D}{P\,w} \qquad (9)$$

$$de = ac\,\frac{sin.}{2}\,2\,\phi \qquad (10)$$

$$w^\circ = \frac{180\,D}{P} \qquad (11)$$

$$E = C\,P\,n\,S\,\frac{sin.}{2}\,2\,\phi \qquad (12)$$

$$l = \frac{\pi\,r\,w}{360} \qquad (13)$$

$$l = \frac{\pi\,r^2}{p} \qquad (14)$$

$$p = \frac{\pi\,r^2}{l} \qquad (15)$$

$$p = \frac{360\,r}{w} \qquad (16)$$

C = centrifugal force in pounds.
B = weight of the revolving body in pounds.
h = velocity of B in feet.
n = number of revolutions per minute.
r = radii of the circle in which B revolves.
v = inclination of the spiral to the radii r.
w = angle in which the spiral is contained.
D = diameter of the propeller.
P = pitch of the propeller.
S = slip of the propeller.

ϕ = angle of inclination of the section of the propeller-blades to the axis.

de = the force which acts to propel the vessel, and caused by the centrifugal force ac.

E = the mechanical effect of de.

We will here add a few formulæ for calculating the pitch and length of an arithmetical spiral. The letters will denote—

l = length of the spiral within the angle $w°$ and radii r.

p = pitch of the spiral, which is equal to the radii r when $w = 360$.

SUGGESTIONS

IN

RELATION TO THE MANNER OF CONSTRUCTING A PROPELLER,

AND THE

ARRANGEMENT OF ITS STEAM-ENGINE, WHEN THE DIMENSIONS OF THE VESSEL ARE GIVEN.

The Propeller.

When a propeller-vessel is to be built, we first have the dimensions of *tonnage, length,* and *beam.* From them the ship-builder lays out the lines of the vessel, and ascertains the tonnage, greatest immerse section area of the displacement, draught of water, &c. From these the propeller is to be constructed, and suppose the following to be the dimensions, viz.:—

Length on deck	230 feet.
Beam	35 "
Draught of water from bottom of the keel	15.5 "
Greatest immerse section area . .	486 sq. ft.
Displacement	1646 tons.

Diameter of the propeller.—The diameter depends on the draught of water, and, in this instance, can be made 14 feet when the draft of water is 15.5 feet.

Pitch.—The pitch of a propeller depends on the arrangement of the steam-engine, as if it is with gearing or direct action. Let the number of revolutions of the propeller be n, when the number of revolutions of the steam-engine is n', a proper pitch will be

$$P = 2.5\ D \sqrt{\frac{n'}{n}}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

of which $D =$ diameter, and $P =$ pitch of the propeller in feet. In this we will construct the propeller for a direct-action steam-engine, and take the pitch

$$P = 2\tfrac{1}{2}\ D = 2\tfrac{1}{2} \times 14 = 35 \text{ feet.}$$

The angle V of the propeller-blades at the periphery will be found by the formula

$$cot. V = \frac{P}{\pi D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$cot. V = \frac{35.}{3.14 \times 14} = 0.795 = cot. 51^\circ\ 30'$$

When the angle V is given, the pitch will be

$$P = cot. V \pi D \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The pitch of a propeller can be found, when the length of the circle arc e is given, and will be

$$P = \frac{\pi D L}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which $e =$ length of the circle arc in the angle v at the periphery, in feet.

When the extreme breadth of the propeller-blades is given at the periphery, the pitch will be obtained by

$$P = \frac{\pi D L}{\sqrt{b^2 + L^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which $b =$ extreme breadth of the propeller-blades in feet over the edge.

Length of the Propeller.—The length must be proportioned to the diameter and number of blades. In the propeller we are constructing, we will take five blades, denoted by the letter m; then the proper length L will be,

$$L = sin. V. \ cos. V \frac{\pi D}{m} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

$$L = sin. 51^\circ \ 30' \times cos. 51^\circ \ 30' \times \frac{3.14 \times 14}{5} = 42.5 \text{ ft.}$$

Number of Blades.—The number of blades in a propeller depends on the slip, and angle V, **Fig. 8.** The more slip, and larger angle V, the less the number of blades should be. As to the number of blades in a propeller, it is known that a less number do better, or just as well, as a greater. It may be so in some circumstances, but not always. In canal and tow-boats, where the slip is considerable, a two or three-bladed propeller will, no doubt, do the best execution; but for vessels which are to run fast, and the propeller has a slight slip, a five or six-bladed propeller should give the greatest effect; but in such cases the propeller must be particularly constructed, and the length L, Figs. 5 and 8, be only one-quarter of the diameter. If this is disproportioned, and the blades are wrong, of course there ought to be a less number of them.

When the length of the propeller is given, we have the proper number of blades by solving the formula (3) to

$$m = sin. V\, cos.\, V \frac{\pi\ D}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$m = sin. 51.^\circ\, 30' \times cos. 51.^\circ\, 30' \frac{3.14 \times 14}{4.25} = 5 \text{ blades.}$$

Acting area of the propeller.—The acting area of a propeller depends on the proportion of pitch and diameter, so that the more the pitch the less the acting area will be L in the same diameter; which will be seen in the formulæ

$$A = \frac{2.5\ D^3}{\sqrt{P^2 + \pi^2\, D^2}}, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which A = acting area.

$$A = \frac{2.5 \times 14^3}{\sqrt{35^2 + 3.14^2 \times 14^2}} = 122.5.$$

Slip.—The slip depends on the greatest immerse section area, and form and friction area of the displacement. When brought into action with the acting area of the propeller, it will be found very near by the formulæ

$$S = \frac{\mathfrak{Q}^2 + 2\sqrt[3]{Q^2}}{16\ A}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$S = \frac{486 + 2\sqrt[3]{1646^2}}{16 \times 122.5} = 0.415, \text{ or say 40 per cent.}$$

$\mathfrak{Q}^2$ = 486 square feet, greatest immerse section area.
Q = 1646 tons displacement.
A = 122.5 square feet, acting area (*less* than $0.785\ D^2$).

This is the formula for calculating the slip when the vessel is new-built and not coppered. When the vessel is coppered, and has run for some time, so that the friction area is very smooth, the coefficient 2 will vary to 1, or perhaps less. Within the common or reasonable proportion of vessels and propellers, this formula follows the variations of slip as near as I, until this, have been able to observe, which has been on a great number of vessels. A formula based upon the principle that the acting area of the propeller, and resistant area of the vessel brought into action, and that the resistance to these areas is in proportion as the square of their velocity, will not follow the variation of slip, and is a complicated calculation.* The slip given by this formula (9) means the slip when the vessel is running in still water, or when there is no other force acting with or against the vessel but that of the propeller. The less the acting area is in proportion to the vessel, the greater the slip will be; and, therefore, the slip can be *no* measure of loss of effect. The slip is nearly constant with different velocities until a certain limit; when it exceeds that, the slip will increase by the excess of velocity.

Velocity of the propeller.—The proper velocity of the propeller depends on the *pitch*, *diameter*, and *slip*, and can by them be proportioned to suit the steam-engine. The proper velocity, or number of revolutions per minute = n, in order to give the vessel the highest speed with the greatest economy of power, is

* See bodies in motion in fluid.

$$n = \frac{200}{S\,P}\sqrt{D}\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$n = \frac{200}{0.415 \times 35}\sqrt{14} = 51.3 \text{ revolutions}$$

per minute.

From this number of revolutions, pitch, slip, and diameter of the propeller, the power of the steam-engine is to be ascertained; but before entering into that, we will finish the propeller. Although it may not be necessary to reach the 51.3 revolutions, but will determine it, for the calculation, to be 50. The propeller to be centripetal, and have an expanding pitch in two directions, as mentioned in the former pages, and first calculate the angle w° from formula 9, page 53.

FIG. 2. FIG. 1. FIG. 3.

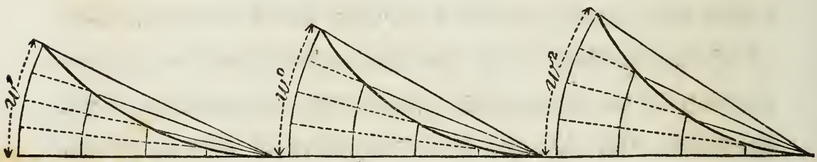


Fig. 1. $$\begin{cases} w^\circ = \dfrac{D\,n^2 S^2}{102.4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11) \\[2ex] w = \dfrac{14 \times 50^2 \times 0.40^2}{102.4} = 54.6^\circ. \end{cases}$$

Compute two more angles ${}^1w$ and ${}^2w$ from the following formulæ:—

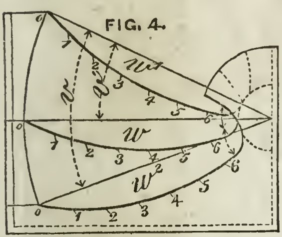
Fig. 2. $$\begin{cases} {}^1w = \dfrac{{}^0w}{2}\,(2—S) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12) \\[2ex] {}^1w = \dfrac{54.6}{2}\,(2—0.40) = \dfrac{54.6 \times 1.60}{2} = 43.8^\circ, \end{cases}$$

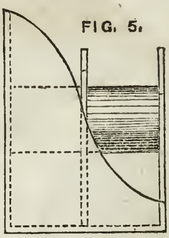
in which angle the spiral is to be constructed, for the fore-edge of the propeller, and Fig. 3 ${}^2w$ is the one for the after-edge.

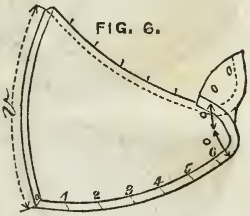
Fig. 3.
$$\begin{cases} {}^2w = \dfrac{{}^0w}{2}\,(2 + S) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13) \\ {}^2w = \dfrac{54.6}{2}\,(2 + 0.40) = \dfrac{54.6 \times 2.4}{2} = 65.5°. \end{cases}$$

The fractions or minutes of those results will not be taken in the construction of the curves.

In each of these angles ${}^0w$ ${}^1w$ ${}^2w$, construct a curved line, as in **Figs. 1, 2,** and **3.**





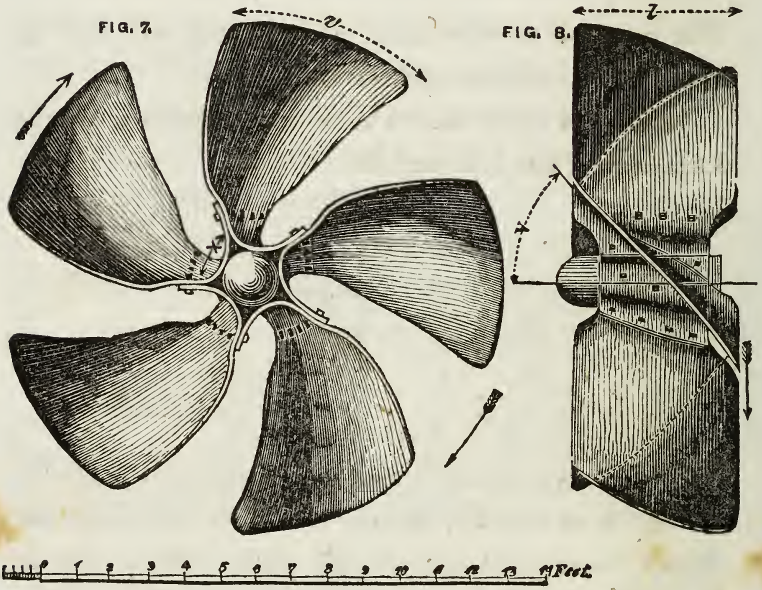


Place those curved lines on a common axis, as shown in **Fig. 4,** so that the planes on which the spirals are drawn be at right angle to the axis. Set the curves ${}^1w$ and ${}^2w$ in a distance equal to the length L of the propeller; then set the spiral ${}^0w$ in the middle, between the planes on which the spirals ${}^1w$ and ${}^2w$ are drawn; draw the straight lines from the centre to the extremity of the spirals. Set the spirals ${}^1w$ and ${}^2w$, so that the angle v will be as follows.

Fig. 4.
$$\begin{cases} v = \dfrac{360\ L}{P} - 0.3\,({}^2w - {}^1w) \quad . \quad . \quad . \quad . \quad (14) \\ v = \dfrac{360 \times 4.25}{35} - 0.3\,(65 - 44) = 37.38°. \end{cases}$$

Place the spiral ${}^0w$ between the spirals ${}^1w$ and ${}^2w$, so that the angles v will be

$$\text{Fig. 4.} \begin{cases} {}^1v = \dfrac{v}{4}\,(2 + S) \quad . \quad . \quad . \quad . \quad . & (15) \\[2ex] {}^1v = \dfrac{37.38}{4}\,(2 + 0.40) = 22^\circ.5. & \end{cases}$$



When the spirals are laid down in these positions, make a drawing of the propeller, as **Figs. 7** and **8**, which represents a propeller made of wrought-iron. If made of cast-iron, the spirals ${}^0w$ ${}^1w$ and ${}^2w$ will be laid out in loom. Build to the spirals a box, as represented in **Figs. 4** and **5**; form it in the hub as in the drawing, so that x is equal to ${}^1x$. In this box, build a pattern of boards, about 2½ by 3 inches; the boards must run from the spiral ${}^1w$ to the spiral ${}^2w$, so that the edges or seams run parallel with each other, or so that the same seam runs to the same figures on the spirals, and forms a regular curve, for instance, a circle arc of a circle, touching the spiral ^0w.

When the slip exceeds 50 per cent., it will be necessary to pay more attention to this curve or circle arc, and lay out 5 w spirals. From this pattern cast a block of cast-iron, represented in **Fig. 6.** Bore holes in this block, corresponding to the holes for the screws, by which the propeller-blades are screwed together. Fit a pasteboard on the face side of the block, and make it the same shape as the propeller-blades are to be; when fitted precisely, mark the holes on the pasteboard; from this, cut out one propeller-blade from the sheet-iron, and bore the holes accurately from the pasteboard. From this blade cut out the rest of the propeller-blades, and one more than the number of blades in the propeller. Afterwards bend these blades over the block (**Fig. 6**) so that the holes correspond with each other, then the blades are ready to screw together. If the holes are not bored before the blades are bent, they must be marked on the block, or else they will not come in their proper position, and it will cause much trouble in putting them together.

If the pitch of the propeller was not expanding in the direction of the radii, the before-mentioned angle v would be from the formulæ

$$v = \frac{360\ L}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

therefore, if we will calculate the mean pitch and angle v at the circumference of the propeller, we have

$$P = \frac{360\ L}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

$$P = \frac{360 \times 4.25}{37.38} = 38.5$$

The Steam-engines.

The effect of a steam-engine sufficient to drive the propeller 50 revolutions per minute will be found by this formula in horse-power = H:—

$$H = \frac{n^2 D^2}{2500} \sqrt{PS} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

$$H = \frac{50^2 \times 14^2}{2500} \sqrt{35 \times 0.40} = 730 \text{ horses.}$$

To be two direct-action condensing-engines, let us determine the effectual pressure per square inch to be 24 pounds; say pressure in the boiler to be 20 pounds; cut off the steam at ⅓d the stroke, the mean pressure will be 14 pounds, vacuum 10 pounds, effectual pressure $10 + 14 = 24$ pounds.

By the quantities we now have given, we will be able to ascertain the size of the steam-cylinders, which may be this:—

$$a = \frac{40\ n\ D^2 \sqrt{PS}}{p\ S\,(1 - f)}; \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

in which a = area of the steam-cylinder in square inches, s = stroke of piston in inches, f = friction and working the pumps per cent., which, in this intended steam-engine, will be $f = 0.32$, or 32 per cent.

Let the proportion of stroke and diameter of the steam-cylinder piston be as 2 : 3, we have the diameter in inches.

$$d = \sqrt[3]{\frac{76\, n\, D^2 \sqrt{PS}}{p\,(1-f)}}, \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$d = \sqrt[3]{\frac{76 \times 50 \times 14^2 \sqrt{35 \times 0.40}}{24\ (1-0.32)}} = 54.75 \text{ inches.}$$

Say 56 inches diameter and 36 inches stroke, the effect of the steam-engine, in horse-power, will be

$$H = \frac{a\, p\, s\, n.4}{33000 \times 12}\,(1-f), \quad . \quad . \quad . \quad . \quad (21)$$

$$H = \frac{2463 \times 24 \times 36 \times 50 \times 4}{33000 \times 12}\,(1-0.32) = 733 \text{ horses.}$$

The number of revolutions that can be obtained by a given power will be found when the dimensions of the propeller and slip are given; or

$$n = \frac{50 \sqrt{H}}{D \sqrt[4]{PS}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

When the dimensions of the cylinders and effectual pressure on the piston are given, the number of revolutions will be

$$n = \frac{d^2\, p\, s\, (1-f)}{51\, D^2 \sqrt{PS}}. \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The effectual pressure which is required to give a propeller so many revolutions per minute will be obtained when the dimensions of the cylinder are given; or

$$p = \frac{51\, n\, D^2 \sqrt{PS}}{d^2\, s\, (1-f)}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Cut off the steam at $\frac{1}{3}$ the stroke, and c being the cubic feet of steam for each number of revolutions in both the cylinders, and k = volume of steam compared with water at the given pressure.

$$c = \frac{2463 \times 12 \times 4}{1785} = 68.4, \text{ say } 70 \text{ cubic feet.}$$

If 1 pound of anthracite coal evaporates 8 pounds of water per hour, the consumption of coal will be, in tons per hour

$$coal = \frac{c\,n}{4.8\,k'}, \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

$$coal = \frac{70 \times 50}{4.8 \times 770} = 0.918 \text{ tons per hour.}$$

The speed of the vessel will be in statute miles per hour,

$$M = \frac{60\,n\,P}{5280}(1 - S) = \frac{n\,P}{88}(1 - S), \quad . \qquad (26)$$

$$M = \frac{50 \times 35}{88}(1 - 0.40) = 12 \text{ miles.}$$

The distance from New York to Liverpool is about 3100 miles; the vessel will run that in a time of $\frac{3100}{24 \times 12} = 11$ days nearly.

To Boston in 10 days, 8 hours. To Halifax in 8 days, 22 hours.

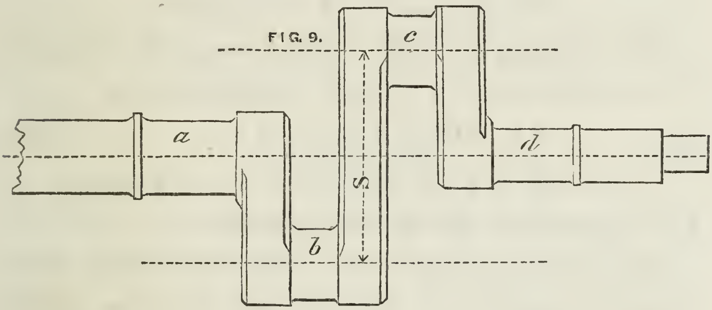
When the power of the steam-engine, dimension, and slip of the propeller are given, to find the speed of the vessel in statute miles per hour,

$$M = \frac{\sqrt{H}\,P(1 - S)}{1.76\,D\sqrt[4]{PS}}, \quad . \quad . \quad . \quad . \quad . \qquad (27)$$

$$S = 1 - \frac{88\,M}{Pn}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (28)$$

The engines are working on a double crank, represented in Fig. 9. The scale is ⅜ inch to a foot. The cranks are opposite to each other; therefore, the steam-engines *do not work at right angles*, but, if it is thought more desirable that they should, the crank can be set in any angle. This is a matter of small consequence, compared with the frictions in the bearings *a* and *d*, which the opposite cranks prevent, and it is the reason why the stroke of the piston can be so short in proportion to the diameter.

FIG. 9.



Now for the strength of this crank, which, at first sight, promises little for durability. Let the end *a* be the continuation of the propeller-shaft, and the steam-engines be applied at *b* and *c*. Let the force which is applied at *c* be denoted by the letter *c*. The force which acts to twist the crank at *b* will be the product of *c s*; but, as there is a bearing at *d*, the resistance in *b* acts to twist the crank at *c* with a force = *b s*. If the crank should break, the force *c* must break it both in *c* and *b* at the same time; which will require twice the power that would break it only at

b, or $\frac{c\,s}{2}$ will be the force by which the crank should be twisted at b, which is exactly the same as if the force c was applied at b to twist the shaft at a; or, if the diameters at a, b, and c be equal, the *strength of the opposite crank* will be the same as a common single crank.

Let **D** be the diameter of the propeller-shaft in inches, and of wrought-iron,

$$\mathbf{D} = 18 \sqrt{PD}, \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\mathbf{D} = 18 \sqrt{35 \times 14} = 12 \text{ inches.}$$

The diameter at d will be the same as the diameter δ of a crank-pin of a common or single crank,

$$\delta = \sqrt{\mathbf{D}^2 + 1.2\, s^2} - 1.1\, s, \quad . \quad . \quad . \quad (30)$$

$$\delta = \sqrt{12^2 + 1.2 \times 3^2} - 1.1 \times 3 = 8.5 \text{ inches;}$$

s is expressed in feet in this equation.

This kind of crank was tried and tested by a Swedish engineer, O. E. Carlsund, and the results were most satisfactory. The engines were 30 inches in diameter, and 18 inches stroke. They made from 105 to 110 revolutions per minute. The air-pumps were direct action, working at the same speed with 18 inches stroke. Pressure in the boiler was 30 pounds per square inch. Cut off the steam at $\frac{1}{3}$ the stroke, the air-pumps and valves worked silently.

In direct-action propeller-engines, the arrangement of the air-pumps and their valves is of the greatest importance. Many engineers are of the opinion that the more vacuum the better; there is, however, an exception with propeller-engines, when the engines and air-pumps are direct action. As an example, in an experiment made by the same engineer, O. E. Carlsund, he placed a small cock on the condenser; while the steam-engine was working with a good vacuum, he opened the cock, letting in a little air to the condenser; the number of revolutions increased from 15 to 20 per cent.

When an air-pump makes more than n double-strokes per minute, it does not work well, or rather works to a disadvantage. This number n will be found by the formula

$$n = \frac{12\,\mathfrak{a}}{\mathfrak{A}\,\mathfrak{S}}\sqrt{\mathfrak{p}}, \quad . \quad . \quad . \quad . \quad (31)$$

in which $\mathfrak{a}$ denotes the area of the valves and $\mathfrak{A}$ = area of air-pump piston. $\mathfrak{S}$ = stroke of the air-pump piston. $\mathfrak{p}$ = pressure in the condenser in pounds per square inch.

When the number of double-strokes per minute are given, the area of the valves will be

$$\mathfrak{a} = \frac{n\,\mathfrak{A}\,\mathfrak{S}}{12\sqrt{\mathfrak{p}}}. \quad . \quad . \quad . \quad . \quad . \quad (32)$$

When we obtain from this formula $\mathfrak{a} = \mathfrak{A}$ or $\mathfrak{a} > \mathfrak{A}$, the air-pump will work with disadvantage; or, when the valve $\mathfrak{a}$ is applied in the air-pump piston, we only get $\mathfrak{a} = \frac{3}{8}\mathfrak{A}$; then $\mathfrak{S}$ and $\mathfrak{p}$ are the only quantities which can be modified, and $\mathfrak{S}$ will be

$$\mathfrak{S} = \frac{12\,\mathfrak{a}}{n\,\mathfrak{A}} \sqrt{\mathfrak{p}}. \quad . \quad . \quad . \quad . \quad . \quad (33)$$

In the steam-engine represented on Plate VIII., the arrangement of the valves is the same as those represented on Plate XII.; the stroke of the air-pump piston is the same as the stroke of the cylinder-piston, and the diameter of the air-pump is 14 inches. In that case, the diameter of the valves must be calculated, and will be as follows:—

$$\mathfrak{d} = \frac{\mathfrak{D}\,\sqrt{n\,\mathfrak{S}}}{3.464\,\sqrt[4]{\mathfrak{p}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$\mathfrak{d} = \frac{14\,\sqrt{50 \times 36}}{3.464\,\sqrt[4]{5}} = 11.5 \text{ inches,}$$

which should be the diameter of the air-pump valves. (See Plate XIII.) These formulæ undergo a little modification with different arrangements of condensers, positions of valves and air-pumps, &c.

The inclination of those engines represented on Plate VIII. is about 16°. It is of no consequence what angle the steam-engines make with each other, but rather make it to suit the bottom of the vessel. There will always be angle enough that one engine will help the other over the centre, and the more it varies from 90° the less friction it will be in the bearings. With that kind of engine, there is sufficient room in the breadth of any vessel to get the connecting-rod twice the stroke. The entire engine comes below the load line, and makes an arrangement with every convenience.

Under full motion, all the parts of the machinery are accessible to the engineer. The weight of both the engines, including air-pumps and condensers, will be—stationary parts 22 tons, moving or working parts 8 tons; total weight 30 tons: occupying a space in the length of the vessel of only 8 feet 4 inches. Lower hold is 12 feet; on the first deck 7 feet; on the second deck 8 feet.

A Table of Equations, collected for convenience and reference.

$$P = 2.5\ D \sqrt{\frac{n'}{n}}. \quad \ldots \quad (1)$$

$$cot.\ V = \frac{P}{\pi\ D}. \quad \ldots \quad (2)$$

$$P = cot.\ V\ \pi\ D \quad \ldots \quad (3)$$

$$P = \frac{\pi\ D\ L}{e}. \quad \ldots \quad (4)$$

$$P = \frac{\pi\ D\ L}{\sqrt{b^2 + L^2}}. \quad \ldots \quad (5)$$

$$L = sin.\ V\ cos.\ V\ \frac{\pi\ D}{m}. \quad \ldots \quad (6)$$

$$m = sin.\ V\ cos.\ V\ \frac{\pi\ D}{L}. \quad \ldots \quad (7)$$

$$A = \frac{2.5\ D^3}{\sqrt{P^2 + \pi^2\ D^2}}. \quad \ldots \quad (8)$$

$$S = \frac{\mathfrak{Q} + 2\ \sqrt[3]{Q^2}}{16\ A}. \quad \ldots \quad (9)$$

$$n = \frac{200}{PS}\ \sqrt{D}. \quad \ldots \quad (10)$$

$$^{\circ}w = \frac{D\ n^2\ S^2}{102.4}. \quad \ldots \quad (11)$$

$$'w = \frac{^{\circ}w}{2}\ (2 - S). \quad \ldots \quad (12)$$

$$^{2}w = \frac{^{0}w}{2}\ (2 + S). \quad \ldots \quad (13)$$

$$v = \frac{360\ L}{P} - 0.3\ (^{2}w - {}^{1}w). \quad (14)$$

$$'v = \frac{v}{4}\ (2 + S). \quad \ldots \quad (15)$$

$$v = \frac{360\ L}{P}. \quad \ldots \quad (16)$$

$$P = \frac{360\ L}{v}. \quad \ldots \quad (17)$$

$$H = \frac{n^2\ D^2}{2500}\sqrt{PS}. \quad \ldots \quad (18)$$

$$a = \frac{40\ n\ D^2\ \sqrt{PS}}{p\ s\ (1 - f)}. \quad \ldots \quad (19)$$

$$d = \sqrt[3]{\frac{76\ n\ D^2\ \sqrt{PS}}{p\ (1 - f)}}. \quad \ldots \quad (20)$$

$$H = \frac{a\ p\ s\ n\ 4}{33000 \times 12}\ (1 - f). \quad (21)$$

$$n = \frac{50\ \sqrt{H}}{D\ \sqrt[4]{PS}}. \quad \ldots \quad (22)$$

$$n = \frac{d^2\ p\ s\ (1 - f)}{51\ D^2\ \sqrt{PS}}. \quad \ldots \quad (23)$$

$$p = \frac{51\ n\ D\ \sqrt{PS}}{d^2\ s\ (1 - f)}. \quad \ldots \quad (24)$$

$$Coal = \frac{c\ n}{4.8\ k}. \quad \ldots \quad (25)$$

$$M = \frac{n\ P}{88}\ (1 - S). \quad \ldots \quad (26)$$

$$M = \frac{\sqrt{H}\ P\ (1 - S)}{1.76\ D\ \sqrt[4]{PS}}. \quad \ldots \quad (27)$$

$$S = I - \frac{88\ M}{P\ n}. \quad \ldots \quad (28)$$

$$\mathsf{D} = 18\ \sqrt{PD}. \quad \ldots \quad (29)$$

$$\delta = \sqrt{\mathsf{D}^2 + 1.2\ s^2} - 1.1\ s. \quad (30)$$

$$n = \frac{10\ \mathfrak{a}}{\mathfrak{A}\ \mathfrak{S}}\ \sqrt{\mathfrak{p}}. \quad \ldots \quad (31)$$

$$\mathfrak{a} = \frac{n\ \mathfrak{A}\ \mathfrak{S}}{10\ \sqrt{\mathfrak{p}}}. \quad \ldots \quad (32)$$

$$\mathfrak{S} = \frac{10\ \mathfrak{a}}{n\ \mathfrak{A}}\ \sqrt{\mathfrak{p}}. \quad \ldots \quad (33)$$

$$\mathfrak{d} = \frac{\mathfrak{D}\ \sqrt{n\ \mathfrak{S}}}{3\ \sqrt[4]{\mathfrak{p}}}. \quad \ldots \quad (34)$$

$$P = \frac{360\ L}{v + (1 - r)\ (^{2}w - {}^{1}w)}. \quad (35)$$

A = acting area of the propeller in square feet.

$$\left.\begin{array}{l} D = \text{diameter} \\ P = \text{pitch} \\ L = \text{length} \end{array}\right\} \text{of the propeller in feet.}$$

S = slip in decimal fractions.

H = Actual horse-power.

M = Statute miles per hour.

Q = displacement in tons.

$\otimes^2$ = greatest inverse section area in square feet.

a = area of the steam-cylinder in square inches.

b = extreme breadth of the propeller-blades, in feet.

c = cubic feet of steam for each revolution.

d = diameter of the steam-cylinder in inches.

e = length of the circle-arc in the angle v in feet.

f = friction and working-pumps per cent.

k = volume of steam compared with water.

m = number of blades in the propeller.

n = number of revolutions per minute.

p = effectual pressure in pounds per square inch.

s = stroke of the piston in inches.

${}^{\circ}w$, ${}'w$, ${}^{2}w$, and v = angle of a circle in degrees.

$\mathfrak{A}$ = area of the air-pump piston.

$\mathfrak{a}$ = area of the valves.

$\mathfrak{D}$ = diameter of the air-pump.

$\mathfrak{d}$ = diameter of the air-pump valves.

$\mathfrak{s}$ = stroke of air-pump piston in inches.

$\mathfrak{p}$ = pressure in the condenser in pound per square inch.

$\mathbf{\mathsf{D}}$ = Diameter of the propeller-shaft.

δ = Diameter of the crank-pin.

Description of the Arrangement of the Steam-Engine and the Vessel.

Plate VIII. represents a section of a vessel of the aforesaid dimensions. On the bottom lie two inclined, direct-action, condensing steam-engines.

A. The engineer's room.

B. First cabin.

C. Ventilators and skylight.

D. State-rooms.

E. Store-rooms.

F. Passages.

G. House on deck.

H. Berths.

The engine is manœuvred in the engine-room, *A*, by the three rods, *a*, *b*, and *c*, which run through the columns in the engineer's room. The four gauges on each side are steam-gauge, vacuum-gauge, water-gauge for the boiler, and salinometer; in the front, is a clock and counter.

a is for regulating the injection-water connected with the injection-cock *k*.

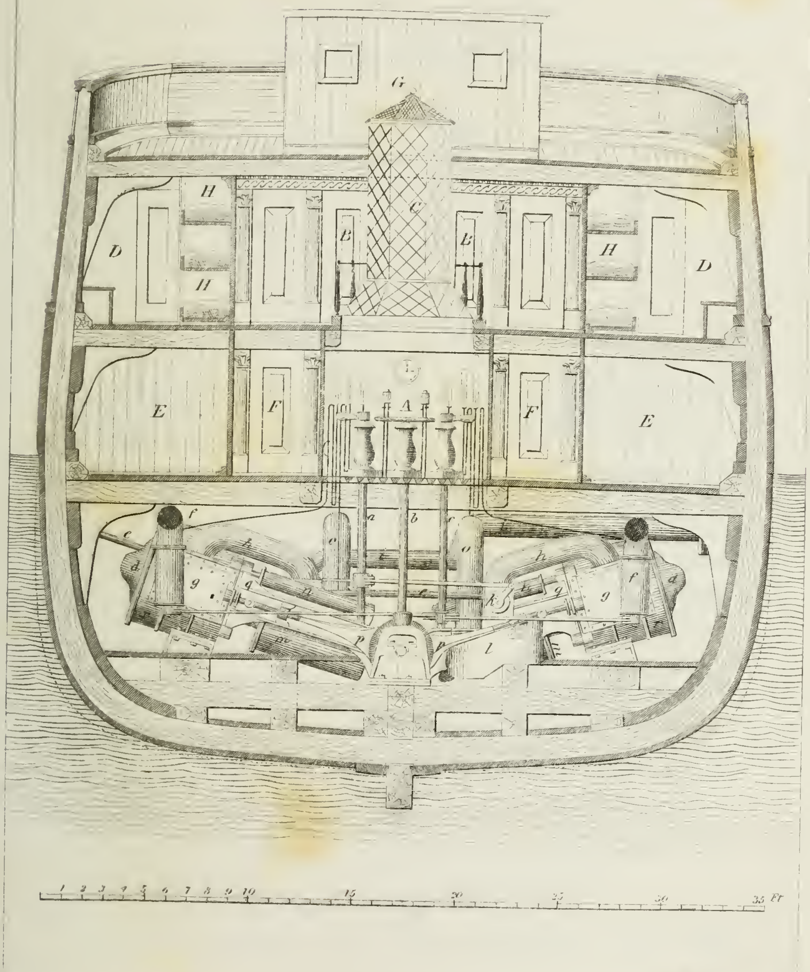
b for reversing the eccentrics, to back and go ahead. Connected with a conical cog-wheel.

c, for regulating the steam to the engine. By the same rod steam can be given to full stroke of the piston.

d. Cylinder-head.

e. Injection water-pipe.

Suggestions for arrangement of Propeller Steam Engines;
by
J.W. NYSTROM.



f. Steam-pipe.
g. Steam-chest.
h. Exhaust-pipe.
i. Discharge-pipe.
k. Injection-cock.
l. Condenser.
m. Air-pump, double and direct action.
n. Feed and force-pumps, double and direct action.
o. Air-vessels.
p. Eccentrics for the steam-valves.
q. Frames and guides.
r. Steam-cylinders.

A Table for finding the pitch of Propellers.

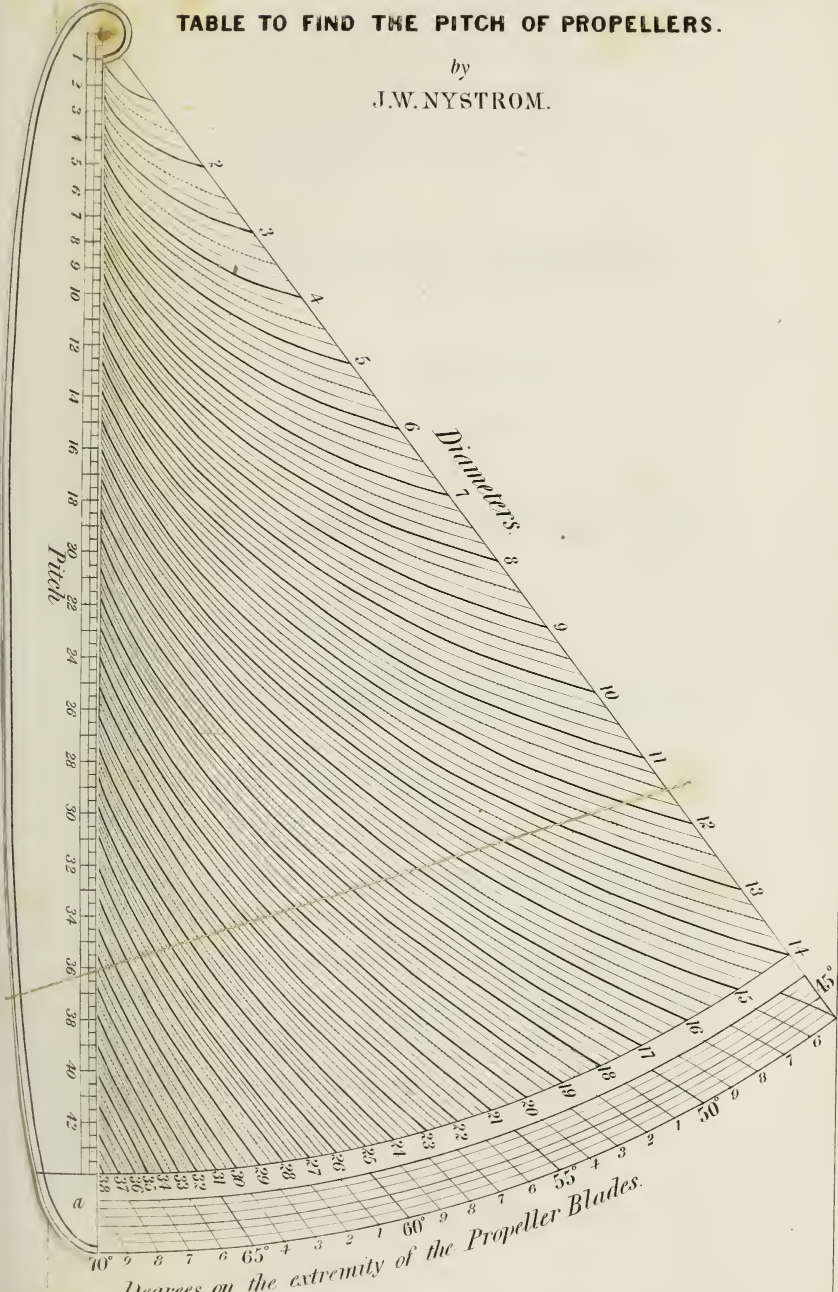
By this table, represented on Plate IX., the pitch of propellers can be found in an instant with any diameter, and angles of the propeller-blades, between 45 and 70 degrees. The degrees will be found on the circular arc, which is divided as a diagonal scale, so that where the arm crosses the diagonals at *a* it shows every tenth minute of a degree.

The pitch will be found on the pitch-arm, where the diameter lines cross the same, when set on a given angle at *a*. If the angle is to be found when the diameter and pitch are given, move the pitch-arm until the given pitch crosses the given diameter line, the corresponding angle will be found at *a*.

Propeller-makers, not fully acquainted with the theory of propellers, will sometimes find it difficult to lay out a propeller with a certain pitch, or when a few diameters of a propeller are given to ascertain the pitch or angle, then, a reference to this table and formulæ will meet their approbation, for convenience in practical use.

TABLE TO FIND THE PITCH OF PROPELLERS.

by
J.W. NYSTROM.



J.W Nystrom, del.

DESCRIPTION

OF A

PROPELLER-ENGINE, PATENTED

BY

R. F. LOPER AND J. W. NYSTROM.

PHILADELPHIA, APRIL 15, 1851.

THE Engine is represented on the Plates X., XI., XII., and XIII.

Fig. 1, side elevation.

Fig. 2, transverse elevation.

Fig. 3, transverse section through the air-pump and condenser.

Fig. 4, the slide-valve motion.

Fig. 5, the air-pump and valves.

Our improvements are particularly applicable to that class of marine steam-engines which are employed to drive screw-propellers, and which it is desirable should occupy the smallest possible space compatible with a due degree of strength and efficient action, and a free access to the several parts for manœuvring adjustments and repairs, where also it is desirable that the engine should work at a high speed, in order to drive the screw-propeller at a sufficient velocity without the employment of gearing, and, at the same time, to obtain a large amount of duty from an engine of comparatively small size and light weight.

The improvements consist, first, in constructing and arranging the condenser and air-pump, which appertain to each steam-cylinder, in such a manner that they shall constitute the columns by which the cylinders are supported.

Second, in the construction and arrangement of the valves to the steam-cylinders working in connection, which is such that the cut-off valve of one cylinder is actuated by the valve-rod, or valve of the other cylinder.

Third, in the construction and arrangement of the air-pump valves, which is peculiarly applicable to engines running at a high velocity, where it is essential that these valves should close quickly.

In the double cylinder propeller-engine, represented in the accompanying plates, it is the bed-frame which is cast hollow, as known in the sections, **Figs. 3** and **5,** and is bolted fast to the kelsons or timbers of the vessel.

The condensers of the two cylinders are mounted upon the one side of the bed-frame, while the air-pumps are mounted upon the opposite side.

The condensers *D* have the form of a hollow cylinder open at the top and bottom; at their lower extremities they are screwed to the bed-frame, and at their upper end are screwed or bolted to the steam-cylinder.

These condensers are each fitted with an adjustage *a*, through which the injection-water is introduced.

The force-pumps *B* are fastened on each side of the condensers, and their plunges are connected direct to the crosshead *E*, and the steam piston-rod *F*. The air-pump column *G*, valve-chest *H*, and side-pipes *I*, are cast in one piece; which is prolonged above the air-pump head, as represented at *b*, formed and fastened to the cylinder and bed-frame, as the condensers on the opposite side. The condenser and air-pumps thus constructed form the columns, on which the cylinders *J* are supported, while the hollow bed-frames form the cistern or well in which the water of injection and condensation collects, and by which they are conducted to the air-pump. In the bed-frame are cast partitions and canals, so that, when the vessel is running in a hard sea, the condensing water will still obtain a regular motion to the air-pump.

Owing to the air-pumps being *double* and *direct action*, the condensing water will be forced out through the discharge-pipe *Q* as *uniformly* as it injects through the adjustage *a*, and thereby render the condensation and operations even. The form of the bed-frame will, however, depend on the form of and room in the vessel.

The steam-cylinders *J* are mounted on the columns *D* and *G*; they have also cast fast to them an exhaust-pipe *V*, through which the steam passes, from one cylinder through the exhaust-pipe *e* in the other cylinder, so that the exhaust steam from each cylinder passes to both the condensers, which make the condensation more even, and an easy passage for the steam from the cylinders.

Fig. 3, the steam approaches the injection-water at *c*, above which is placed a cock, by which the steam can be let out through the pipe *U*, and the engine will work with high pressure.

Fig. 4, each cylinder is fitted with a piston *M*, piston-rod *F*, and crosshead *E*, and connecting-rod *L*, which latter takes hold of a pin of a crank *N*, secured to a crank-shaft *X* beneath. The crosshead is steadied by guides secured to the inner faces of the condenser and air-pump.

Each cylinder is fitted with an appropriate steam slide-valve *P*, which is moved by an appropriate eccentric, secured to the crank-shaft of the engine, and traverses upon the face in which the steam and exhaust parts *f*, *e*, are formed.

In each steam-valve are formed the suitable passages *d*, *d*, *g*, which, by the motion of the valves, are made to move over the steam and exhaust part of the valve-seat.

The face of each valve slides upon a valve-seat, while the back of each valve forms a seat, to which a supplementary slide-valve *h* is fitted, by whose movement the steam passages *d*, *d*, of the steam slide-

Plate X

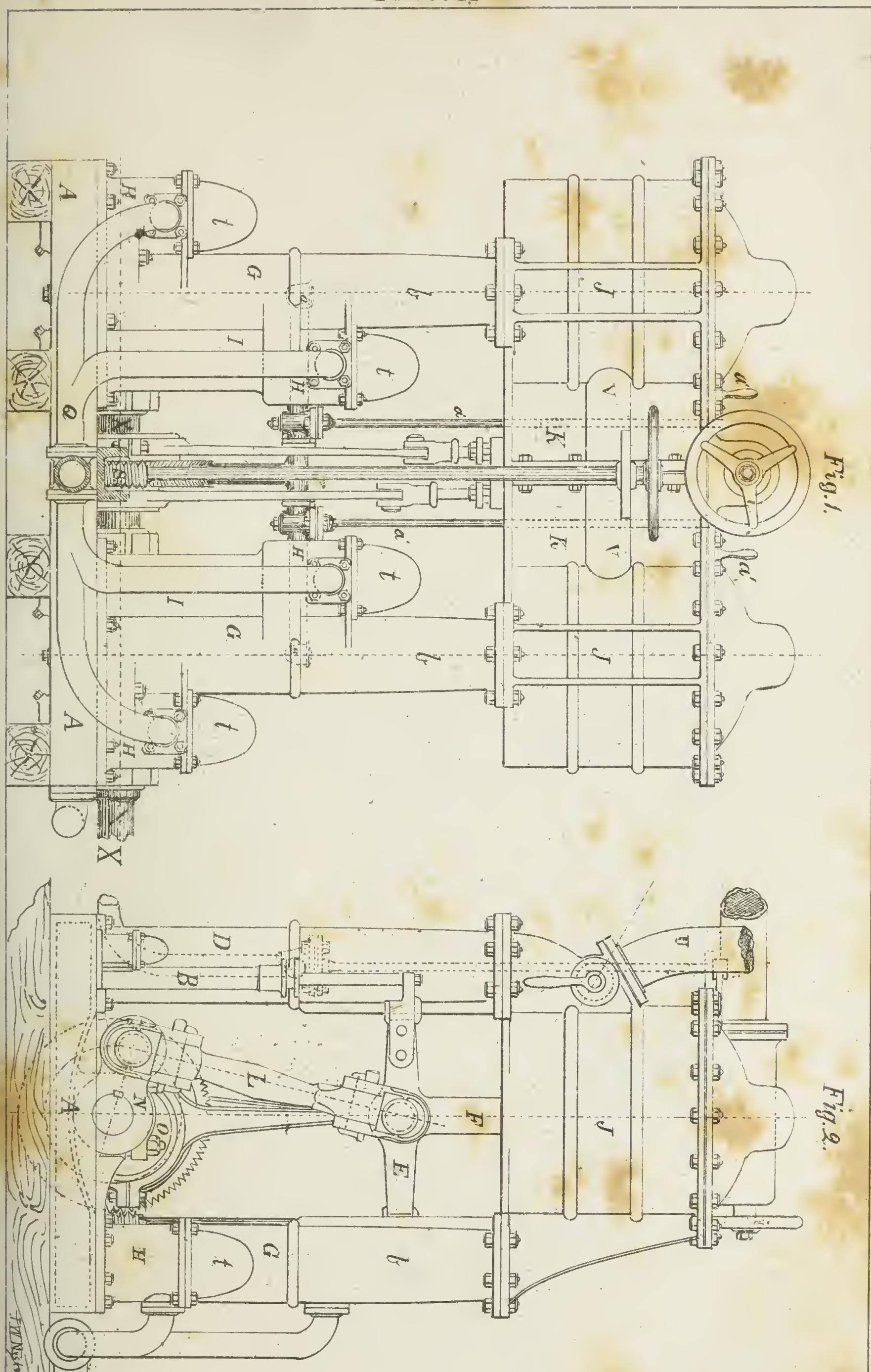
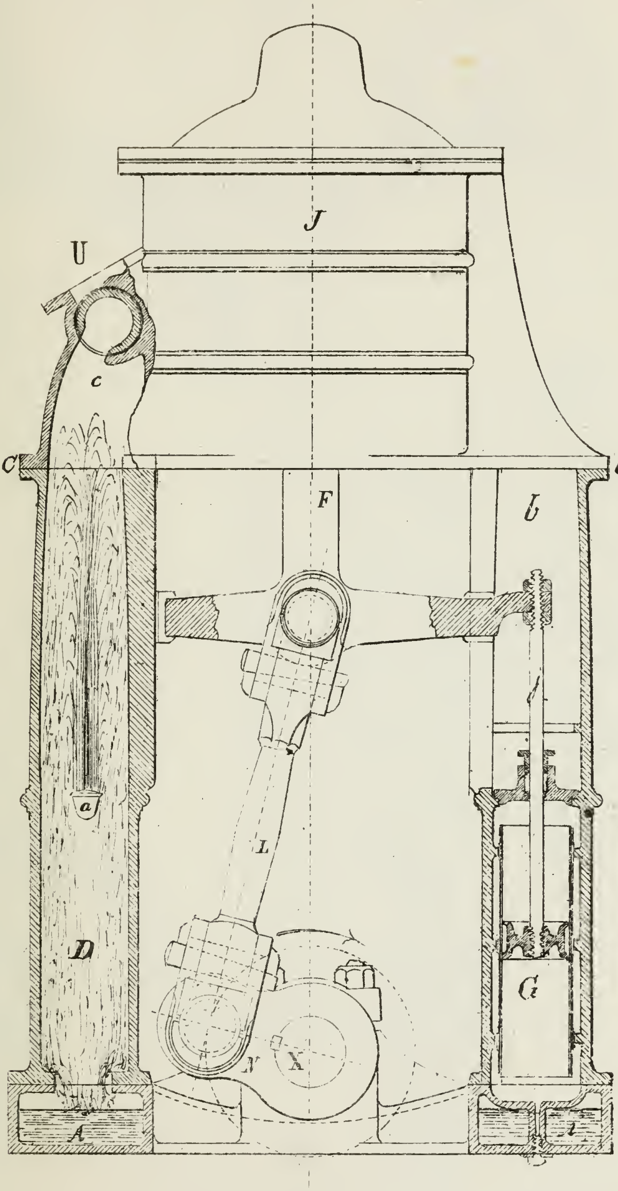


Fig. 1.

Fig. 2.

Fig 3.



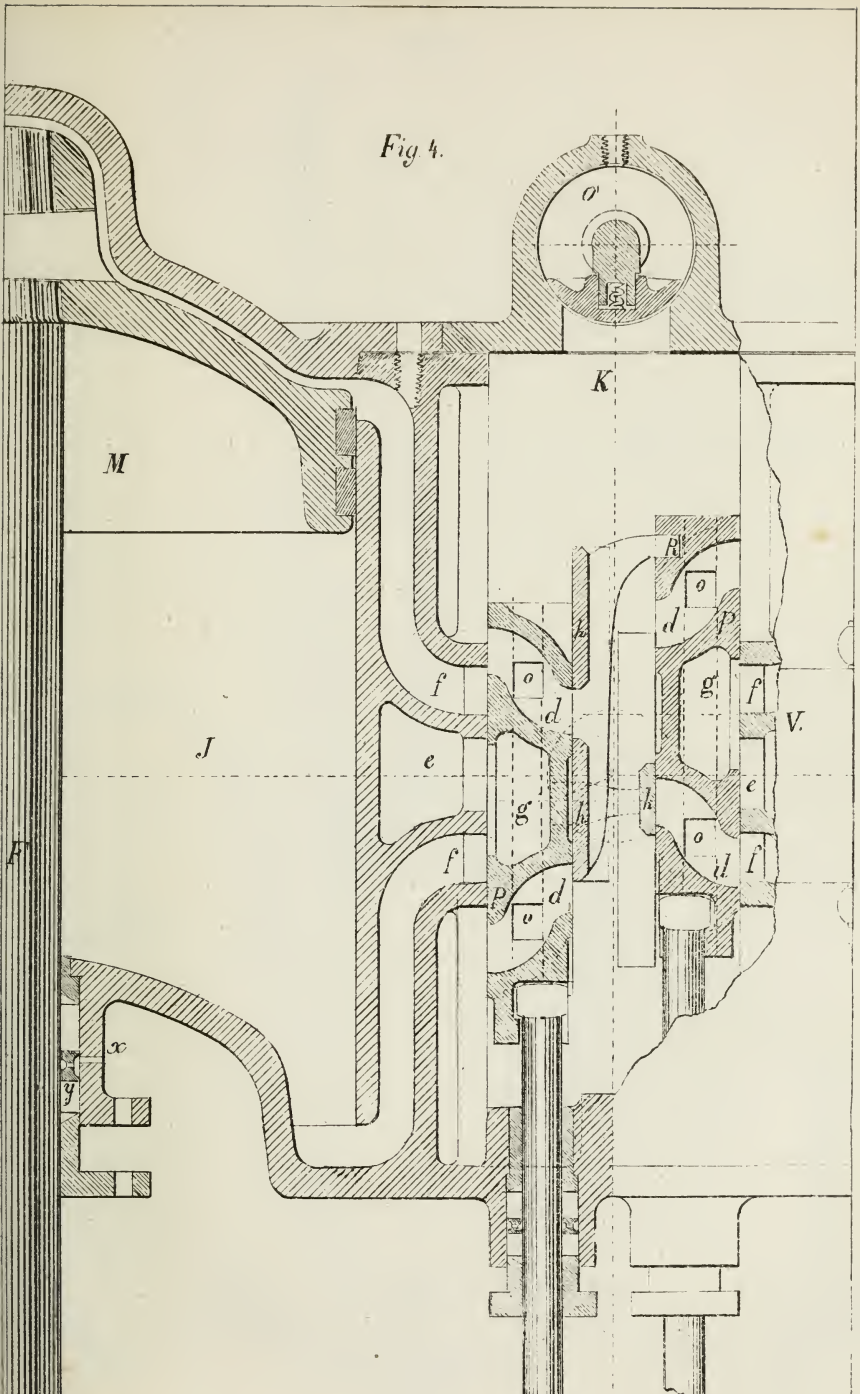


Fig. 4.
O'
K
M
R
o
d
P
h
g
f
V.
e
J
k
F
x
y
J.W. Nystrom.

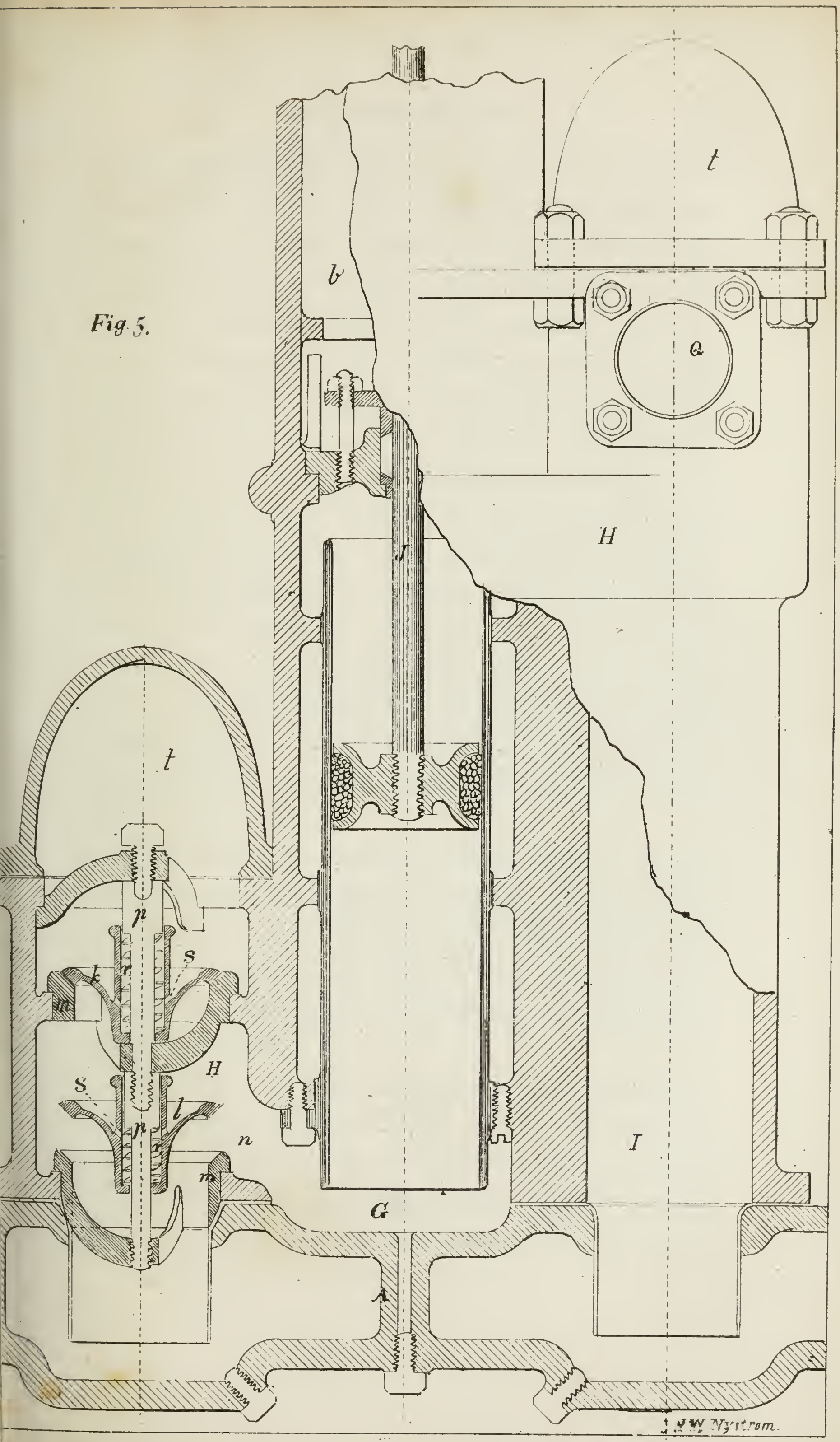


Fig. 5.
b
t
Q
H
J
t
p
r
k
s
m
H
S
l
p
r
n
m
G
A
I
J. W. Nystrom.

valves are closed at the proper moment, to cut off the admission of steam to the cylinder. The supplementary or cut-off valve of each cylinder is connected by a link, and moved with the steam slide-valve of the opposite cylinder; and as, in this example, the cranks and eccentrics of the two cylinders are set at right angles with each other, the cut-off valve is moved with the greatest speed, while its appropriate slide-valve is moving with less speed, or is almost stationary.

The eccentrics are fastened to the cog-wheel, which is geared by the screw *S* (see *Manœuvring the Engine*) in such a position that the *mean cut-off* will be about one-third of the stroke; but, if it is desirous to admit more steam, or to *cut-off* at a greater part of the stroke, it can be regulated by fastening the eccentrics to the cog-wheel in such a position as the cut-off requires, which can easily be regulated between $\frac{1}{3}$d and $\frac{5}{8}$ths of the stroke.

This method of arranging and working the valves is extremely advantageous in cutting off the steam, as the operation is effected with rapidity, while it is peculiarly applicable to fast-moving engines, where it is desirable to employ the least complex and the most simple machinery.

When the engine is to be manœuvred to go back or ahead, and the steam is cut off at $\frac{1}{3}$ of the stroke, it may happen that the engines stand in such a position that no steam can be admitted to the cylinders, which obstructs the starting of the vessel; for that purpose, there are apertures, *o, o, o, o,* in the side of the steam-valve, which can be opened and closed at pleasure, with a suitable slide-valve connected with a rod which runs through a stuffing-box on the steam-chest, and thereby the steam can be let into the cylinders at full stroke, which renders the starting safe at any position of the engine. When the engine is to be worked at full-stroke of steam, these supplementary valves are adjusted by the engineer in such a position, that the apertures *o, o* are uncovered throughout the whole stroke, thus permitting the steam to enter through these apertures when those at the back of the steam-valves are closed by the cut-off valve *b*.

Each valve-chest contains two valves, *k* and *l*, and their appropriate valve-seal *m*; each communicates with the adjacent air-pump barrel by a passage *n*, which leads from the space between the two valves. The lower valve-chest is connected directly with the hollow bed-frame, while the upper is connected therewith by means of a pipe *I*, through which the water passes to the lower valve of the chest.

The valves are conical or trumpet shaped, in order that they may open without a shock when the water strikes them; and each is secured to the stem, which traverses upon a stationary spindle *p*. The prompt closing of each valve is insured by a spiral spring *r*, coiled upon the stationary spindle, and concealed within the tubular stem, which also protects it from injury. As the air-pump piston is moved up and down in its barrel, the lower valve l, of the valve-chest rises alternately, to admit the water to pass into the air-pump barrel, while the opposite upper valve of each chest rises correspondingly, to allow the water to pass to the discharge-pipe Q.

As the space in which the spring is closed is alternately diminished and increased in capacity by the opening and closing of the valves, it is essential that free passage should be given, to allow the water, which may be drawn therein when the valve drops, to pass out freely as the valve opens; this is effected by perforating the tubular stems with small holes represented at s, **Fig. 5.**

As the crosshead has the air-pump to work on one end, it is of importance to arrange it so that the friction in the guides will be on the opposite end, where also the force-pumps are connected, so that the work on both ends of the crosshead will be nearly equal; this is obtained by moving the engine in the direction of the arrow, **Fig. 3,** which will be understood by examining the figures 2 and 3.

Manœuvring the Engine.

To reverse the eccentrics, is operated by an endless screw S, geared into a wheel, partly movable on the shaft X, to which wheel the eccentrics are fastened.

The screw S has a V tread, and so arranged that, if the engine should start while the screw is in gear, it will throw itself out of gear.

The steam is regulated by the wheel *i* (**Fig. 1**), connected with the valve *o′* (**Fig. 4**). The injection-water is regulated by the handle *a′*, connected with the rod *a*, injection-cock, and adjustage *a*.

The air-pump of the engine, represented in the accompanying Plate XIII., is double and direct action. The piston-rod *j* is connected direct to the extremities of the crosshead of the steam piston-rod, so that, when the latter moves up and down, the air-pump piston is moved in a corresponding manner. Each air-pump barrel is fitted with two valve-chests *H H*, one near the bed-frame, the other at the same height as the injection-cock. See **Fig. 1.**

Economy of space is one of the most important requisites in steamers; and it is believed that an engine of this arrangement and construction attains this requisition in a higher degree than any upright condensing engine now in use.

While it excels in this respect, it is evident that such an engine will not weigh as much as others whose members are arranged in the usual manner; for in this engine the ordinary framing for supporting the steam-cylinders is dispensed with, and the condenser and air-pumps are made to supply their place, as well as to their own peculiar duty. In the condensation of steam, and the production of vacuum, this economy of weight, in connection with the corresponding economy of space, is all important; as any saving in the weight and space occupied by the machinery increases in a corresponding manner the capacity of the vessel for carrying freight.

What we claim as our invention, and secure by letter patent, is, First, the construction and arrangement of the columns, by which the steam-cylinder is connected with the bed-frame in such a manner that they constitute the air-pump and condenser, substantially as herein set forth.

Second, the method herein described, of actuating the cut-off valve of one steam-cylinder, by a motion derived from the valve or valve-rod of the other cylinder, substantially as herein set forth.

Third, the adjustable supplementary valve *o*, in connection with the aperture *o*, in the steam-valves, by means of which the steam can be worked at full pressure, throughout the whole length of the stroke without disengaging the cut-off valve.

LOPER'S PROPELLER.

Plate XIV. represents the one known as Loper's Propeller. Its peculiarity consists in that the propeller-blades form an angle with the centre line in the centre of the same, and, therefore, is no screw.

Loper's rule for this angle is to make it from 25° to 45° at the hub, independent of the diameter of the hub, or proportion of pitch and diameter of the propeller. That this angle really constitutes a novelty of the instrument is thoroughly tested.

The one represented on Plate XIV. is the original propeller on the steamer S. S. Lewis, and is a true representation of Captain Loper's own design, with the following dimensions:—

Diameter	13 feet.
Length	3′ 8″ inches.
Mean pitch	33 feet.
Angle of the blades of the extremity	50°
" " hub .	27°
The propeller was geared to run .	$1\frac{3}{4}$ turn.
while the steam-engine made .	1 "

Dimensions of the Steam-Engine.

Diameter of cylinders	60 inches.
Stroke of the piston	40 "
Steam pressure per square inch .	20 pounds.
Cut off the steam at one-half the stroke, vacuum about . . .	10 "
Leaving an effectual pressure per square inch on the piston . .	26 "
which gave the propeller revolutions per minute	44

Dimensions of the Vessel.

Length on deck	210 feet.
Beam	30 "
Draft of water to the 7th water-line	15 "
Greatest immerse section area .	400 sqr. feet.
Area of 7th water-line . .	2250 "
Tonnage of displacement .	1293 tons.

The steamer S. S. Lewis left Philadelphia for Boston, September 13, 1851.

Passed the Philadelphia navy yard at	10h. 24′
" Brandywine light-house .	6h. 25′
a distance of 90 miles in . . .	8h. 1′

in which time the steam-engine made 12,270 revolutions; multiplied by 1.75, will be 214,745 revolutions of the propeller, and travelling

$$\frac{214{,}745 \times 33}{5280} = 134 \text{ miles,}$$

of which will be a

$$\text{slip} = \frac{134-90}{134} = 32.85 \text{ per cent.}$$

The wind was south-west, and the sails were set in about $\frac{3}{8}$ths of the distance, which diminishes the slip somewhat.

By the formulæ 8 and 9, the slip would be

$$S = \frac{2.5}{13^3} \sqrt{3.14^2 \times 13^2 + 33^2}^{\,2} (400 + \sqrt[3]{1293^2})$$

$$= \frac{2.5 \times 52.45 \times 5164}{13^3} = 30.8,$$

which was the slip first calculated. Her speed in statute miles per hour will be

$$M = \frac{90}{8\text{h } 1'} = 11.2 \text{ miles.}$$

Her speed, first calculated, was from the formula (26)

$$M = \frac{40 \times 33}{88} (1-0.308) = 10.33 \text{ miles per hour;}$$

with the supposition that the steam-engine should be large enough to transmit a power of 350 horses, clear to the propeller, by which we calculated the number of revolutions of the propeller by the formula (22): or

$$n = \frac{50 \sqrt{350}}{13 \sqrt[3]{33 \times 0.308}} = \frac{50 \times 18.7}{13 \times 1.78} = 40 \text{ revolutions}$$

nearly.

The S. S. Lewis was intended to run between Boston and Liverpool, but made only one trip. In a

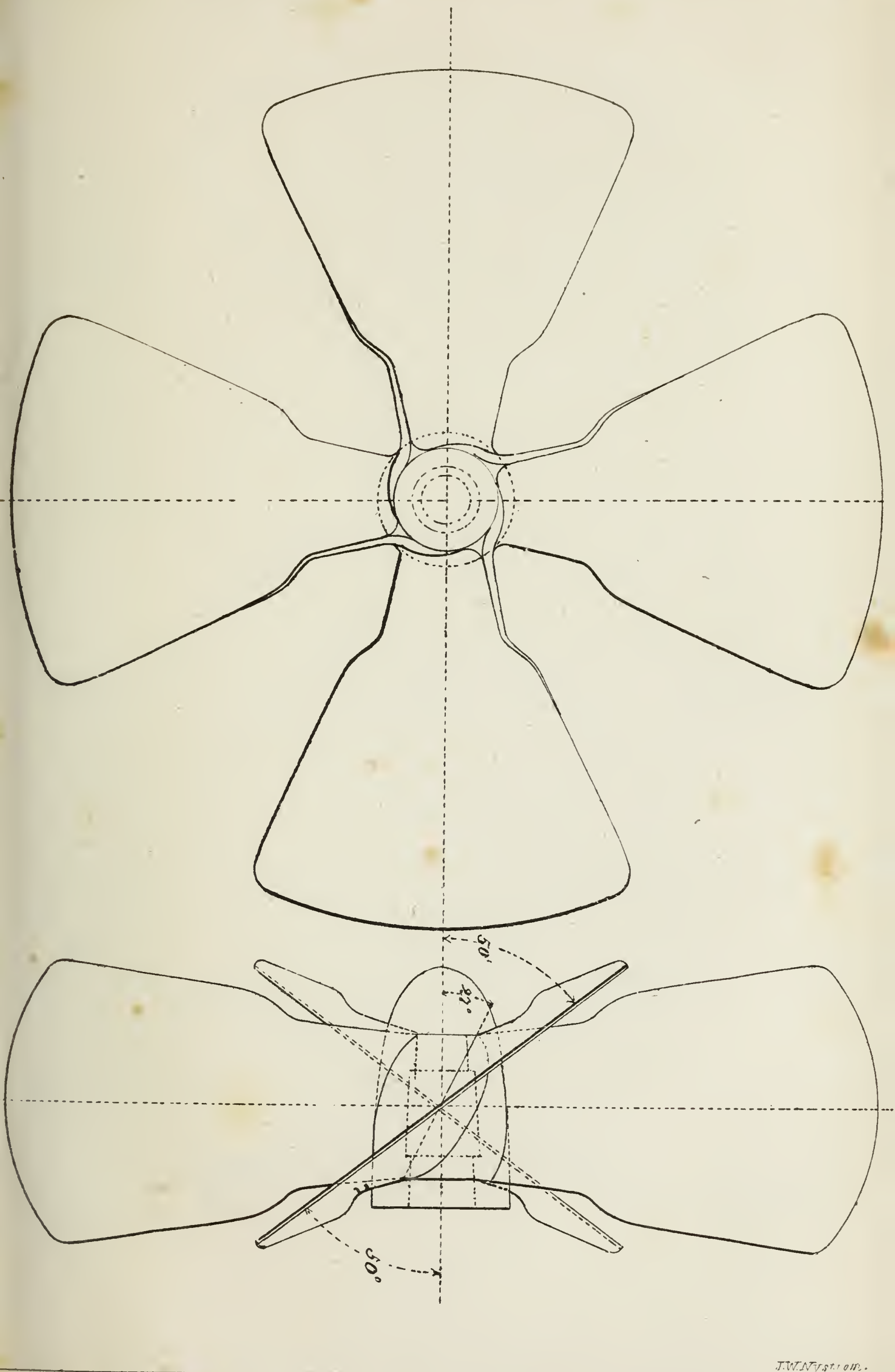
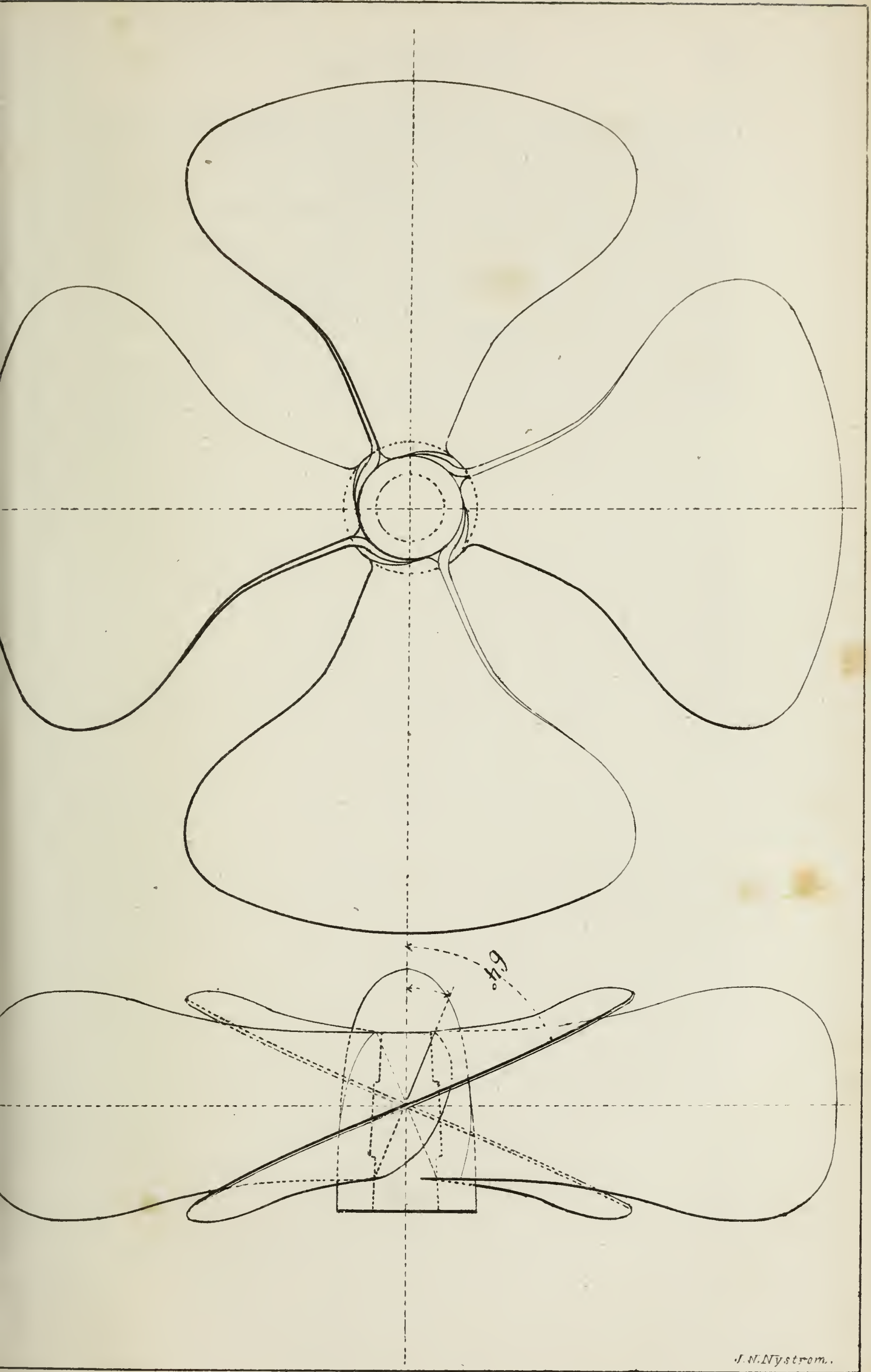

50°
27°
50°
J.W.Nystrom.

Plate XV.



J. N. Nystrom.

distance of 2600 miles from Boston, which she made in 11 days, the propeller broke, and she was obliged to finish the remaining 300 miles under sail. In Liverpool, another propeller was put on, which differed from Loper's propeller—first, in that it was a regular screw; second, it had 39 per cent. less pitch than the former one. Plate XV. is a true representation of the propeller made in Liverpool. Its pitch is only 20 feet.

A CENTRIPETAL PROPELLER.

PLATE XVI. represents a centripetal propeller constructed on the principle mentioned on page 60, that the curvature of the generatrix is independent of the slip, or velocity of the propeller; but that the inclined generatrix has an inclination to the centre line about 45°, and the angle w°, calculated from the formula (11) page 61, and the mean pitch at the extremity of the propeller-blades being about 2.5, the diameter, we have

$$w^\circ = \frac{180\,D}{2.5} = 72^\circ. \quad . \quad . \quad . \qquad (11)$$

At the same time the propeller is constructed with an expanding pitch in two directions, as mentioned on page 60. The expansion in the direction parallel to the centre line, is measured by the angle V of the screw helix to the centre line, in the point where the pitch is to be calculated. In Fig. 2, Plate XVI., the line $a\ b$ represents the helix of the expanding screw, at the extremity of the propeller-blades. It will be found that the angle $V = 64^\circ$ at the point a at the fore-edge of the propeller, which will be a pitch of

$$P = cot.\ 64^\circ \times 3.14\ D = 1.48\ D,$$

but in the after-edge, at the point b, the angle $V = 46°$, and pitch $P = \cot. 46° \times 3.14\ D = 2.93\ D$; then, the expansion of pitch at the extremity of the blades will be as

$$1.48 : 2.93 = 11 : 21 \text{ nearly as } 1 : 2;$$

but this is not the true expansion of the screw, which depends on the second expansion from the centre line, to the extremity of the blades, measured by the difference of the angles w and w'. See page 53, about those angles.

It is evident that the angles w and w' must be proportioned by the mean pitch in the fore and after-edge of the propeller, taken at a diameter $= 0.7\ D$.

Let us, without any calculation or formula, determine the angles

$$w = 58°$$
$$w' = 78,$$

which is in the neighborhood of 72 degrees given by the formula (11) page 61; then, at any distance $= r$ from the centre, the mean pitch will be found by the formula (35),

$$P = \frac{360\ L}{v + (1 - r)\ (w' - w)}; \quad . \quad . \quad . \quad (35)$$

in which

L = length of the propeller, which, in this instance, is $\frac{1}{3}$ of the diameter.

v = the angle in which the propeller-blades are projected at the periphery, which, in this instance, is 52°.

r = a fraction of the radii of the propeller.

Suppose the centre of effort of the propeller-blades is at 0.7 from the centre, we have the mean pitch for the propeller

$$P = \frac{360 \times \frac{1}{3} D}{52 + (1 - 0.7)(78 - 58)} = 2.07 D.$$

The pitch in the centre of the propeller will be

$$P = \frac{360 \times \frac{1}{3} D}{52 + (1 - 0)(78 - 58)} = 1.666 D.$$

The true expansion of the mean pitch will be calculated as follows:—

in the fore-edge

mean pitch $P = (1.666 - 1.48)(1 - 0.7) + 1.482 = 1.537$,

in the after-edge

mean pitch $P = (2.93 - 1.66)\, 0.7 + 1.66 = 2.55$;

of which we obtain the true expansion within the screw to be as

$$1.537 : 2.55 = 6 : 10 \text{ nearly.}$$

Suppose this propeller to be applied on a vessel with similar dimensions as the S. S. Lewis, and having a diameter $= 13$ feet; then, from the foregoing, we will obtain the following pitches:—

$P = 19.25$ feet in the corner *a*.

$P = 38$ feet in the corner *b*.

$P = 21.55$ feet in the centre.

$P = 26.9$ feet, the *mean pitch of the propeller*.

$P = 20$ " " " fore-edge.

$P = 33$ " " " after-edge.

A propeller constructed on this principle, with such arrangement of expanding pitch, will act to propel

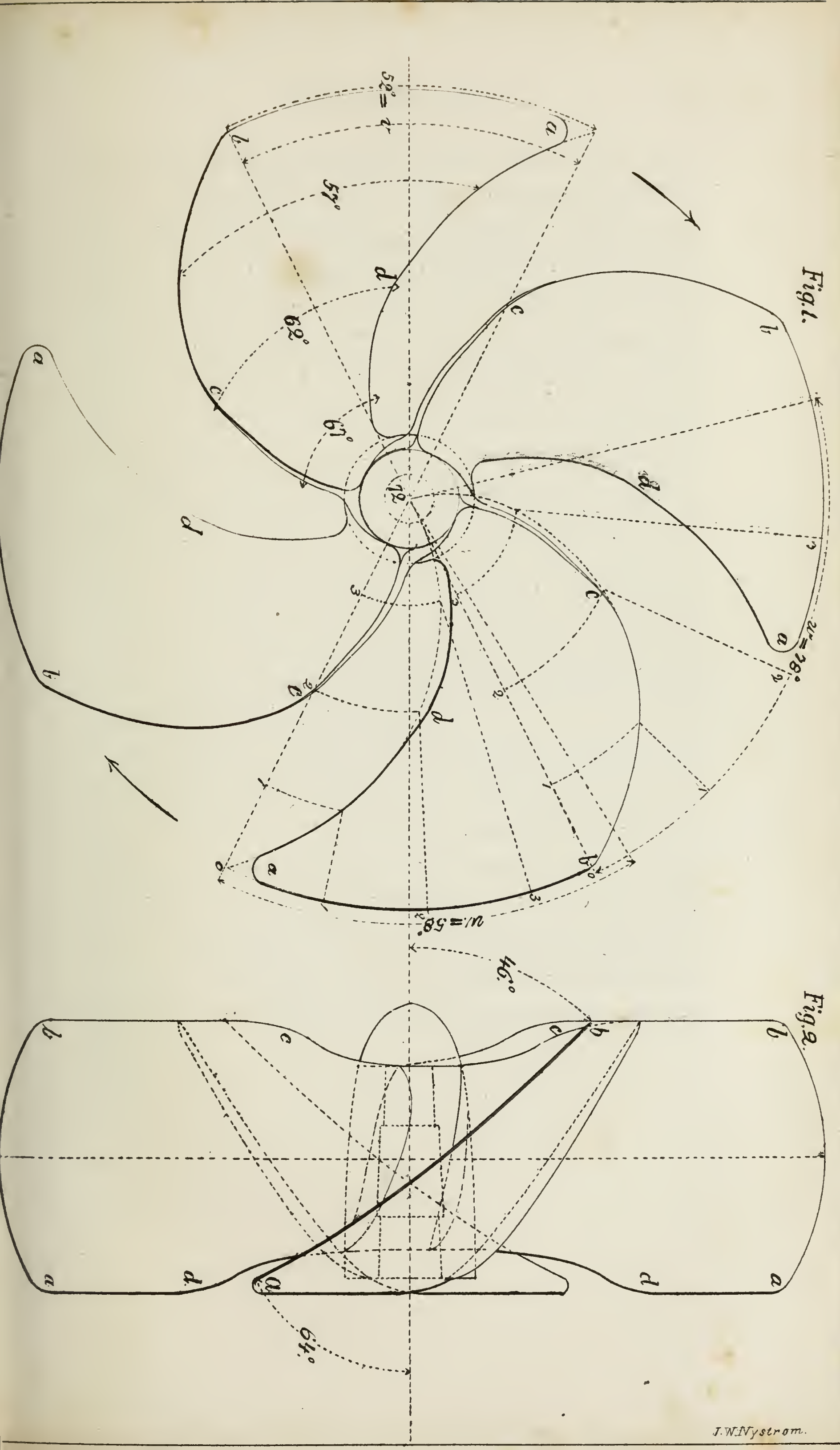


Fig. 1.
52° = a
57°
62°
67°
w = 28°
u = 58°
Fig. 2.
46°
64°
J. W. Nystrom.

the vessel with the greatest effect within the limits of 70 and 20 per cent. slip; and the propeller making from 40 to 60 revolutions per minute. That is to say, it will act to propel the vessel with its greatest effect within the limit of 15 and 3 miles per hour, caused by fair and head wind, of which the 3 miles should be against a gale. We do not mean to say that those (15 and 3) are standard points, but do say that it is impossible to reach that limit by a propeller with a uniform pitch. It is often found that propellers with more pitch, even in a moderate head-wind, lose some of their power; and, on the other hand—as the English propellers—with a very narrow pitch, when running in a fair wind, the propeller is often of no use for propelling, and sometimes has a negative slip, which keeps the vessel back.

The English propellers now running between the United States and England, do not expect to make what is termed a very good time, something like 9 or 10 days; but, even if they have head-wind the whole trip, they will come *surely* in not far from their usual time. That the average time for Atlantic propellers of more or less pitch will probably be in favor of the latter, provided they are constructed as usual into the day.

IMPROVEMENTS

OF

STEAM-ENGINES AND PROPELLERS.*

BY

O. E. CARLSUND.

THE inventions and improvements granted by this letter patent consist in, that by means of propellers, and thereto suitable machinery, propel as well merchant vessels as men-of-war, principally with those direct-action steam-engines, *obtaining the greatest possible effect with the greatest economy of fuel, space, and expense of labor.*

Description.

Plate XVII., **Fig. 1,** shows a vertical section of a steam-vessel, with its (in angle) direct-action steam-engine. Plate XVIII., **Fig. 2,** shows the plane of the same engine. **Figs. 3** and **4** are details of the same. Same letters refer to the same parts. The principle on which the machinery acts is as follows, viz.: *a* is

* Translated from O. E. Carlsund's Patent (Swedish).—N.

the propeller-shaft on which the crank *b* is applied. On this crank are applied two direct-action steam-engines, which form an angle with each other, so that, when one of them stands on the centre, the other one acts to turn the crank, and thereby obtain the rotary motion. The proper angle which the machinery ought to form with each other, I have found to be between 90° and 128°; but it can without detriment be made more or less. The advantage which is gained by machinery of this description is, that it makes a simple and compact engine, which occupies the least possible space in the length of the vessel. It also allows a sharp section area of the vessel, and thereby diminishes the resistance area of the same. It is in this manner to place machinery in vessels in which my principal invention consists, which was accomplished by me in 1843—and I have since then, made several improvements, that have been applied to a great number of propel-vessels; which detailed improvements I will here describe.

To work this engine direct, and without gearing, with the velocity which is required for propellers, it has been of great importance to diminish the moving parts, weight, and dynamic momentum, for which I have applied a new sort of piston, *c*, **Fig. 1**, which consists of a strong plate of a concave form, with iron packing rings. This piston is fastened to a piston-rod *d*, which outer end is guided in a bearing and a connecting-rod *e*, combine the piston-rod direct with the crank. The air-pump f is directly combined with the piston-rod by the crosshead g, also the force-pump h, and thereby simplifies the motion, and, by the air and force-pump's resistance, diminishes and balances the machine's momentum and friction in the bearings. In consequence of the velocity and great number of revolutions per minute which this direct-acting machinery must make, it demands a quite different construction of the air-pumps and their valves than those which have been applied in engines for paddle-wheels or propeller-engines with gearing, because otherwise the valves could not sustain the violence of the water caused by its inertia. It has, therefore, been of importance to construct these pumps and valves to overcome the mentioned difficulties, and has, by several experiments, so succeeded that there is machinery in this country whose air-pumps make a velocity from 300 to 400 feet per minute, and empty the condensing water from 70 to 120 times per minute. The manner in which I have succeeded to produce such a result is as follows:—

The air-pump f (Plate XVII., Fig. 1) is *single and downwards acting.* The water and air which this pump, while it goes up, has taken in from the condenser k, through the bottom valve l, presses while it goes down through the top valve m, in the time piston presses against the layer of air, which, when going up, has interspaces; which layer of air forms an elastic packing between the air-pump and the water, which

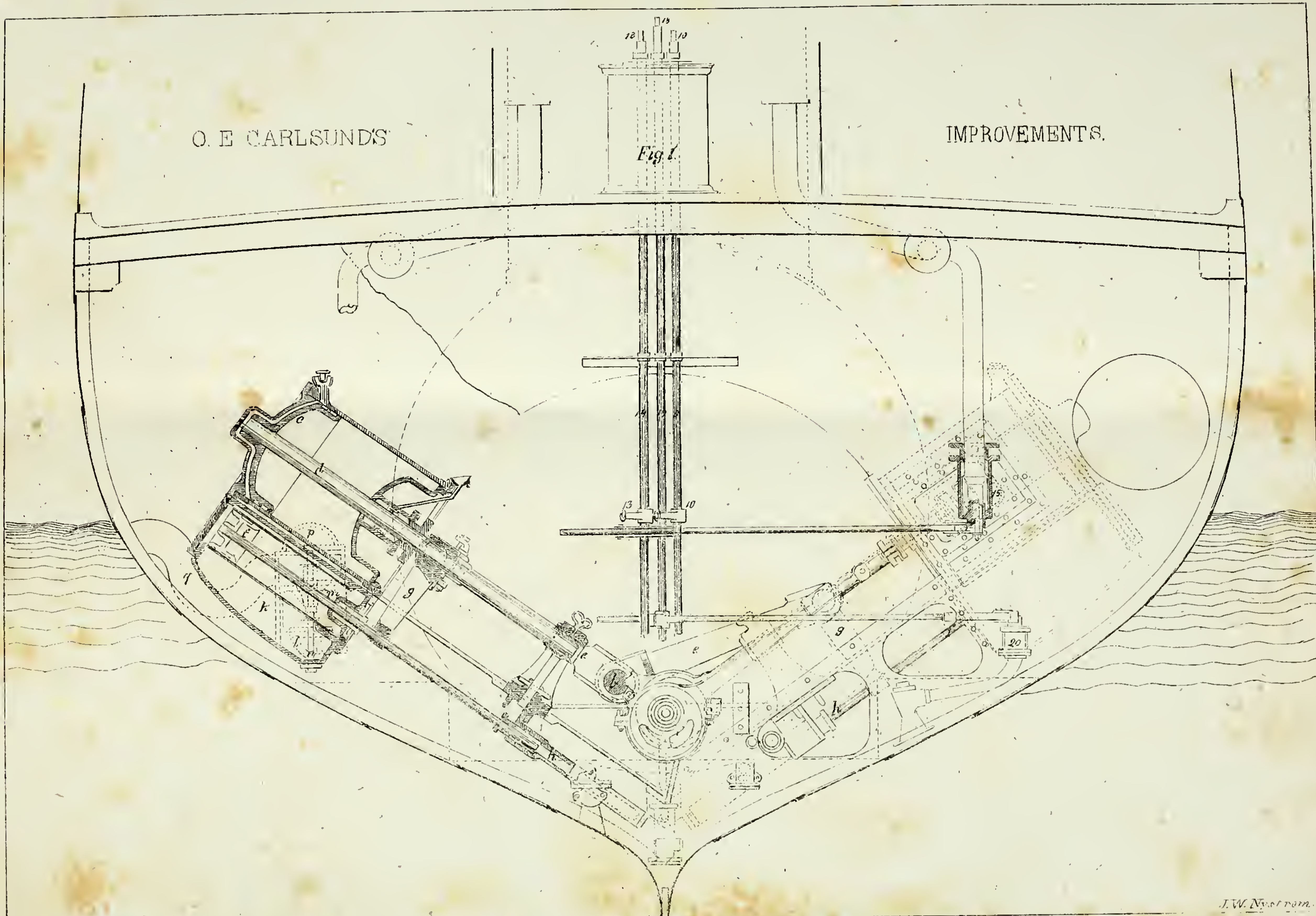


Plate XVII.

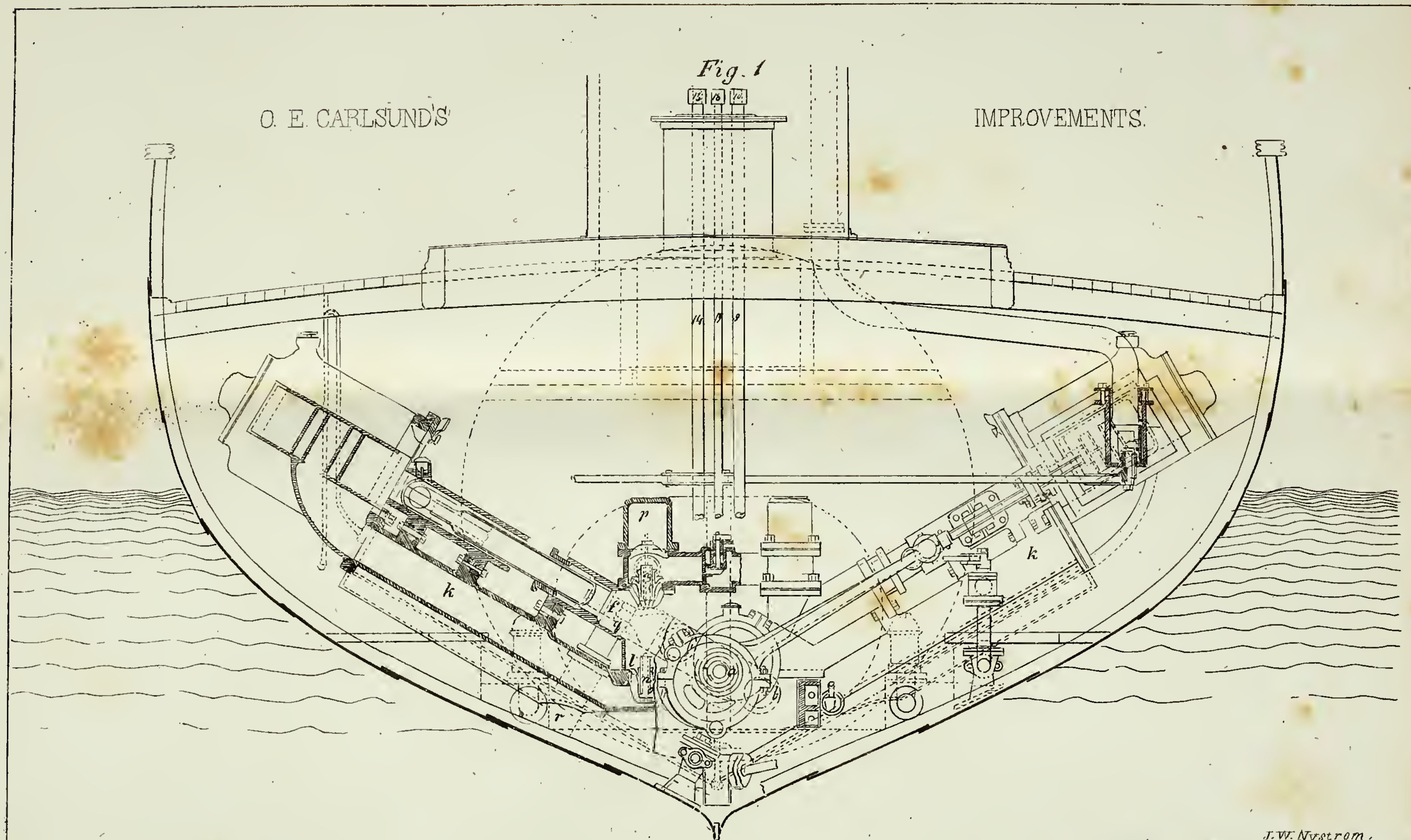


Plate XIX.

is to pass through the top valve *m*, and thereby damp the shock of the piston against the bottom. Further, that the valves *l* and *m* may obtain a slow motion, it has a *conic or parabolic form*, whereby the water presses on it increasing, until it lifts up the same; this form is also suitable to the free motion of the water. Notwithstanding this form is suitable for the quick motion of the valves, there is applied a spring of brass, or any other suitable material, around the spindle *n* on which the valves move. This spring prevents the violent shock which otherwise would occur; and, when the piston returns, the spring presses the valve down quicker than it would do of its own weight and acceleration.

Further, I have applied a reservoir *p* over the top valve with a discharge-pipe *q*. This discharging-pipe has a crooked form by which the water will meet an elastic medium, and thereby uniform the violent motion.

This is the principle which constitutes my invention of air-pumps and their valves, and has been applied on a number of vessels. The machinery is manœuvred as well on the deck as in the engine-room.

Plates XVII. and XVIII. show in detail the manner in which this engine and air-pumps were built in Motala machine-shop (Sweden) in 1843. In the propeller-shaft, *a* is a hole in which is a round bolt 1, through which inner edge is a square rod 2, which latter can, when the bolt 1 moves in or out,

8

move itself into a groove 3, on each side of the shaft *a*. This rod 2 projects over the shaft *a*, and forms a screw, corresponding to the screw-nuts 4 and 5, on which the eccentrics 6 and 7 are fastened. The bolt 1 obtains an alternative motion from the cog-wheel 8, and the screws give the eccentrics a rotary motion on the shaft *a*, thereby regulating the motion of the steam-engine to *stop*, *back*, and *ahead*. This cog-wheel 8 gets its motion by the rod 9, which can be manœuvred by the engineer on deck, or in the engine-room by the handle 10. In the same manner, the engineer can manœuvre the steam and injection-water by the rods 14 and 19, as follows:—

The rod 14 is combined with the steam-valve 15, by which the steam can be regulated at pleasure, to the steam-chest through the canals 17 and 15, through which the steam can be admitted at full stroke or cut-off. The rod 19 combines the injection-cock 20, and regulates by the handle 18. This simple mechanism answers all the purposes that are necessary to manœuvre the engine, and, when the steam is worked with great expansion, it makes the manœuvring sure at any position of the engine, as the steam can be let through the canal 15 direct to the inner steam-valve, and, when the engine is started by moving the handle 15 a little, the canal 15 will be shut, and the engine work as usual with cut-off.

Plate XIX. represents a vertical section of a steam-boat and *engine* (*in angle*) *direct action*. Plate XX. shows the plane of the same engine. This engine is a later improvement on the one before described. It was accomplished in 1848, and will be described as follows:—

The propeller-shaft *a* is formed of opposite cranks *b b*, on which the connecting-rods are applied, each on its crank, so that when the crank stands, say vertically, the one engine pulls or pushes on its crank just as the other one pulls or pushes in the opposite direction on its crank, and thereby balance each other, from which arise the following important advantages (see Tables on pages 32, 33, &c.).

1. *The two engines balance each other* so that no counterbalance is required, and thereby a greater regularity in the moving system is obtained.

2. *The powers applied to the crank counteract each other*, so that the greater part of the friction in the bearing is dispensed with, which is inevitable with the single crank.

3. *That the two engines' dynamic momentum suppress each other*, that the shaking and side motion of the vessel is thereby damped.

The consequence of these advantages is that machinery of this construction can, without any increased friction, have a considerable shorter stroke in proportion to the diameter, and thereby obtain a greater number of revolutions than heretofore has been possible. Also, a greater expansion of the steam can be used, from which a greater economy of fuel is obtained. It is not necessary that these cranks should be precisely opposite each other; they can be set in any angle. Also, the machinery; it is not necessary it should have the same angle as on the drawing.

The condenser *k* also constitutes the framing and bed-plate for the engine, which makes a strong, compact, and simple machinery, in which are saved weight and expense. The air-pumps and also the force-pumps on this engine are of a different construction than the one before described; but their principle is the same, because they are single and downward acting, and accompanied with the same parabolical valves as before described. The pumps press the water and air through the discharge-pipe *q*, which is fastened to the bottom of the vessel. The machinery of this construction has proved, to the most favorable advantage, that it makes from 105 to 110 revolutions per minute. The air-pumps and their valves, and the whole machinery work silently and even; the condensation uncommonly good. Machinery of this construction occupies the least possible room in proportion to its power.

Plate XXI. **Fig. 1** is a sectional drawing of a *merchant steam-ship, of a larger size, with a horizontal direct-action steam-engine.* **Fig. 2** shows the plane of the same. **Fig. 3** is a section of details.

This kind of engines cannot be placed in so sharp vessels as those before described, but are more suitable for vessels of larger size. The principle of the engine, and its operation will be described as follows:—

To the piston c are annexed two piston-rods d, which run over the shaft a, on both sides of the cranks b, b, to the crosshead g, which is guided in the frames. To this crosshead is directly attached the air-pump f, force-pump h, and connecting-rod i, to the cranks b b. The condenser k, reservoir p, and air-pumps, also the framing and bearings, are all cast in one piece. The *air-pumps and force-pumps are double action.* Their valves are of the same construction as before described; the water is forced out through the discharge-pipe q, which runs through the bottom of the vessel. (See Plate.) The one end of the force-pumps feeds the boiler with water, and the other end for launching the vessel, and other purposes.

Horizontal direct-action engines have in other countries before been used, but the air-pumps and force-pumps have not been attached direct to the crosshead, but have been geared from cog-wheels to obtain the *slow* motion. It is in this manner to apply air- and force-pumps direct from the crosshead, and to combine the condenser air-pumps and framing in one piece, which constitute my invention. One engine of this kind has been accomplished by me.

Plate XXII. **Fig. 1** is a section of a man-of-war with a horizontal direct-action steam-engine of 300 horse-power. **Fig. 4** shows the plane of the same.

This kind of engines are more suitable for men-of-war, because the engine comes entirely below the load-line, and, therefore, inexplorable for hostile shots. This engine differs from the former one in that it has four piston-rods *d d d d*, of which one lays over and one under the propeller-shaft on each side of the cranks *b b*. The crosshead *c*, which combines the four piston-rods, moves in the same plane as the propeller-shaft, which was not the case in the former one with two piston-rods, where the connecting-rod formed a greater angle to the centre line in its lower, than in its upper, position. In this engine, those angles will be equal. The condenser, air-pump, and framing are all cast in one piece, as described in the former one, but the framing on one side of the engine contains the double-acting force-pumps cast in one piece. In the former engine, the cylinders laid on one side of the vessel, but in this one, the cylinders lay zigzag. **Fig. 2** shows the double-acting force-pump *h*, with its valves *l* and *m*, canals for the water, and air-vessels. **Fig. 3** shows the air-pump *f*, with its valves *l* and *m*, cistern *p*, and discharge-pipe *q*, in the framing and condenser *k*. The valves here described will be seen on Plate XIII. on a larger scale.*

Fig. 4 shows a new stuffing-box for horizontal and inclined piston-rods or shafts, &c. This stuffing differs from the common by the ring *S*, which separates the packing into two parts. This ring has two

* These valves are as near Carlsund as I can remember them.—N.

grooves turned, on the *in* and *outside*, to leave room for the tallow and oil which is let into the cock. In the grooves are bored a number of holes, through which the oil or tallow runs into the piston-rod or shaft, and they being constantly oiled and clean. (The stuffing-box is shown on a larger scale, on Plate XII.)

Plate XVIII. **Fig. 6** shows the plane and elevation of my invented propellers. By experience and calculations it has been confirmed that the common propellers of Ericsson, Smith, Hunter, &c., also the several different kinds of propellers which I have tried* in this country, all labor under two essential disadvantages, viz.:—

1st. That the water, by the centrifugal force, is thrown out to the periphery in the direction of the radii, in proportion as the velocity of the propeller increases, from which results a loss of effect in propelling the vessel. This loss of effect increases as the resistance of the vessel increases, as in head-winds and towing, especially when the propeller has a large pitch in proportion to the diameter.

And 2d. That the propeller-blades on a common propeller are too much exposed to braching, when the vessel is running in harbors and shallow water, or in narrow canals.

These essential defects are remedied by the propeller which is here described.

Fig. 6, *a*, the propeller-shaft; *b*, its centre, to which a

* Carlsund did not try any with curved generatrix.—N.

number of blades, *c c c c c*, are fastened. These blades can be placed to form a regular screw, or rather with an expanding pitch, so that the fore-edge of the blades only cut the water while the vessel is running its uniform speed. This expanding pitch gives the propeller-blades a parabolical form, so that, when operating, it gives the water an accelerative motion backwards, until its resistance balances the velocity and resistance of the vessel forwards. The ring or band *d* which circumscribes and combines the propeller-blades, I have generally made the same breadth as the length of the propeller in the direction of its centre line, but it can, without detriment, be made more or less. It is not necessary that this band shall be cylindrical; it can have a slight conical form. Also, it is not necessary that this band shall be fastened to the propeller-blades; it can be stationary to the vessel, and the propeller revolves in it.*

This outer band has the advantage of counteracting the centrifugal force of the water, but throws the water backwards parallel to the centre line of the propeller, so that, when the propeller runs in shallow water on canals, it does not interrupt the bottom or sides of the same. That this is a fact is thoroughly tested in the canal between Stockholm and Gothenburg. The vibration and shaking which the common propeller gives to the vessel are, by this band, entirely

* Inventions of this kind have been patented in this country (America), but have not succeeded.—N.

O. E. CARLSUND'S IMPROVEMENTS

Fig. 1.

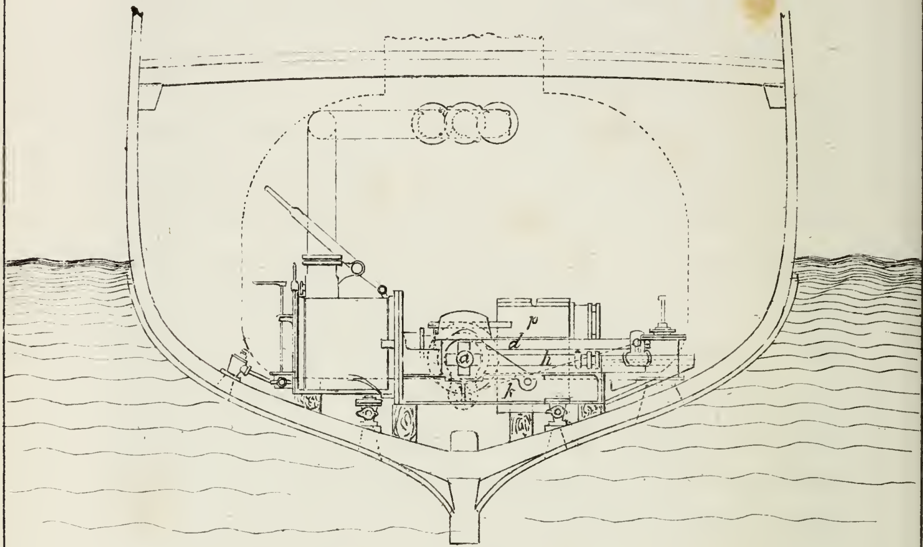


Fig. 2.

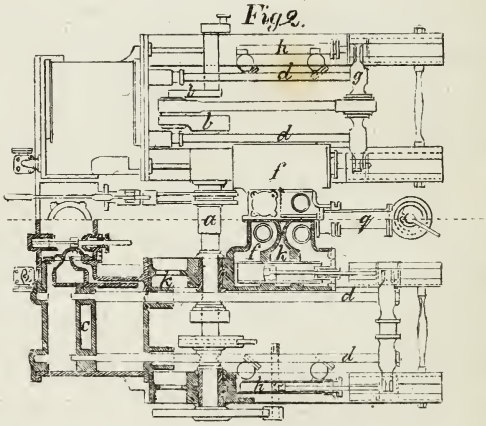
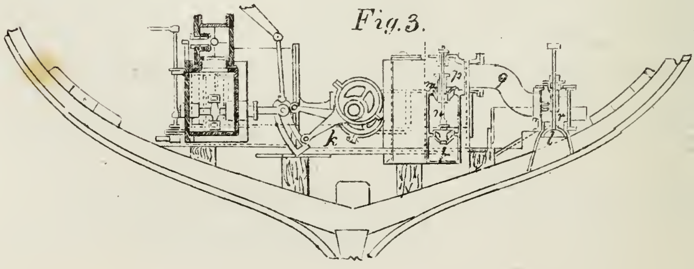


Fig. 3.



J. W. Nystrom.

O.E. CARLSUND'S IMPROVEMENTS.

Fig. 1.

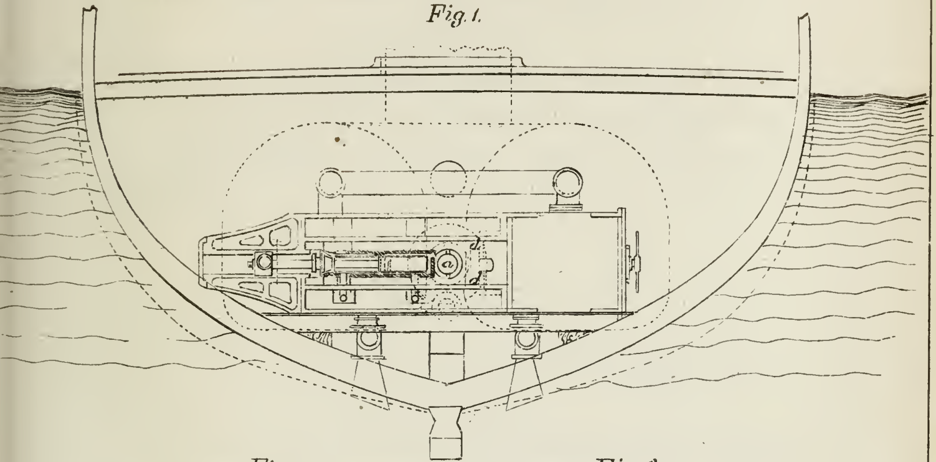


Fig. 2.

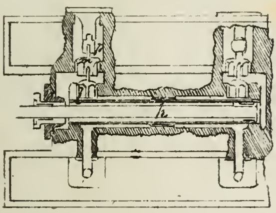


Fig. 3

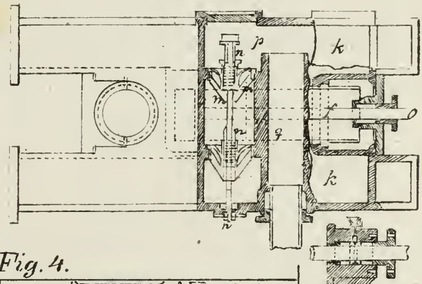
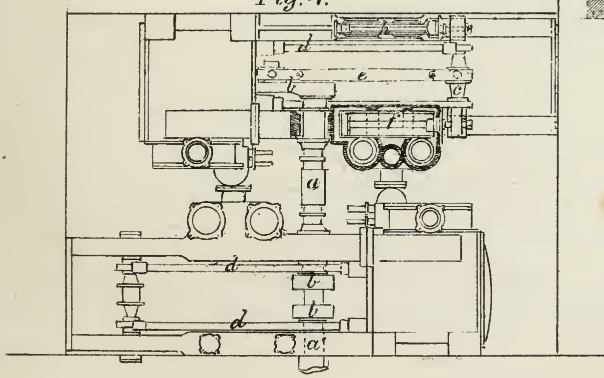


Fig. 4.



J.W. Nystrom.

avoided; and the band acts as a fly-wheel to govern the motion of the machinery, which is of importance when the engine works with great expansion of the steam.

This propeller has been applied on several vessels, and has proved the above-described advantages, that I hereupon declare the following claims to be my invention:—

Claims.

1st. The in angle direct-action steam-engine, working on the same crank and shaft.

2d. The in angle direct-action steam-engine working on two opposite cranks on the same shaft.

3d. To apply direct to the crosshead the air-pumps and force-pumps, and to use the conical or parabolical valves moving on stationary spindles with spiral springs in the closed stem, and the downwards-running discharge-pipe for the condensing water.

4th. To manœuvre the engine on deck, or in the engine room, by means of revolving the eccentrix as herein described.

5th. To use the bed-plate as condenser.

6th. The horizontal direct-action steam-engines with one or more piston-rods attach the air-pumps and force-pumps direct to the crosshead of the engine, and to use the condenser as framing for the engine.

7th. To use the outer band around the propeller-blades.

A TREATISE

ON

BODIES IN MOTION IN FLUID;

WITH

PRACTICAL RULES, AND EXAMPLES HOW TO CALCULATE THE RESISTANCE FOR ANY DESCRIPTION OF BODIES.

RESISTANCE to bodies in motion in fluid is a subject which has, from an early period, received attention by experiments, but the philosophy of it has not yet been presented in a well-established theory. The latest and most extensive experiments have been accomplished by MORIN, in France, LAGERHJELM, in Sweden, and BEAUFOY, in England.

Those experiments have been very extensive, but incomplete for the purpose they were intended; that, even at the present time, comparatively little light has been thrown on it from experiments or theory.

In the years 1811–12–13–14 and 15, extensive experiments were made in Sweden, conducted under the sanction, and at the expense of the Society of Iron-Masters at Stockholm, by Messrs. LAGERHJELM, FORSELLES, and KALLSTENIUS. The experiments were accomplished at the Fahlu Mine, and published in two volumes, of which Assessor LAGERHJELM sent some copies of the first volume to Colonel BEAUFOY (in the year 1819), who was deeply interested in the subject; Colonel Beaufoy experimented, and published a work at his own expense, entitled "Nautical and Hydraulic Experiments," London, 1834.

It is not to be expected that, in so small a compass as this book, will be found a complete work on a subject embracing the amount of matter that bodies in motion in fluid does; and it is a subject which particularly belongs to ship-building. This is a book devoted to screw-propellers, and the arrangement of their steam-engines; yet, the two are so connected in navigation, that it will not be deemed out of place here to glance at it.

A plane A immersed in fluid (**Fig. 1**, Plate XXIII.) will sustain a hydraulic pressure on each side equal to the weight of a column of the fluid, with the same base as the plane A, and an altitude equal to the depth d of the centre of the plane under the surface of the fluid.

Letters will denote:—

$A =$ area of the plane immersed; in square feet.

$d =$ depth of the centre of gravity of the plane below the surface of the fluid; in feet.

$e =$ the weight of one cubic foot of the fluid in which the plane is immersed.

$P =$ hydrostatic pressure in pounds, on one side of the plane, or on the fore side if the plane is in motion.

$p =$ hydrostatic pressure in pounds on the opposite side of P.

If the plane is stationary, it will be $P = p$, and the hydrostatic pressure

$$P = A\,e\,d, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Example 1. **Fig. 1.**

Suppose the plane . $A = 4$ square feet,
immersed to a depth . $d = 3$ feet in fresh water,
we have . . . $e = 63$ pounds.

Require the hydrostatic pressure:—

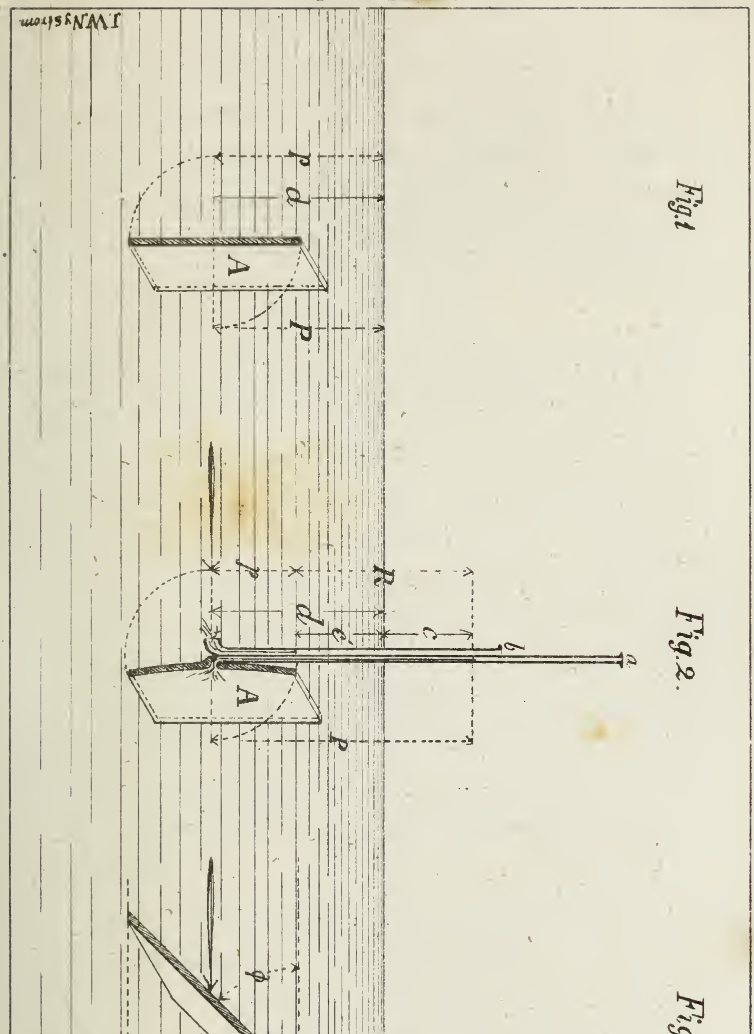
$$P = 4 \times 63 \times 3 = 756 \text{ pounds on each side.}$$

Fig. 2.—If the plane A *moves* in a direction at right angle to itself, the pressure P will increase on the fore-side of the plane, but the pressure p on the opposite side will be diminished.

Suppose a plane $= A$, at a depth $= d$, moves at a velocity $= v$ feet per second. From its centre extend two tubes, a and b, vertical over the surface of the fluid. The tubes to be open at both the ends, but, in the immersed ends, bent at right angle, so that the aperture in the tube a is turned *opposite* the direction of motion, and the aperture of the tube b *with* the direction of motion, as seen in Fig. 2.

Now, if the plane A moves in the direction of the arrow, the water will rise in the tube a and descend in the tube b, so that the *ascension* $=$ *descension* over and under the water line.

Plate XXIII.



Call the ascension $= c$, and descension $= c'$. The height of the column of water which *resists* the motion of the plane will consequently be $P = d + c$, and the height of the column which acts *with* the motion will be $p = d - c'$; then we have for P and p the pressures

$$P = A\,e\,(d + c), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$p = A\,e\,(d - c'), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The force R, which gives the plane the motion, will consequently be the difference of P and p, or

$$R = P - p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Example 2. **Fig. 2.**—Suppose the plane A moves at a velocity $= v$, so that the ascension $c = 1$ foot, and, also $c' = 1$ foot, $A = 3$ square feet, $d = 5$ feet, $e =$ 63 pounds. What will be the two pressures P and p?

$$P = 3 \times 63\,(5 + 1) = 3 \times 63 \times 6 = 830 \text{ pounds},$$

$$p = 3 \times 63\,(5 - 1) = 3 \times 63 \times 4 = 756 \text{ pounds},$$

and the force $R = 830 - 756 = 74$ pounds, which is the force which gives the plane the motion.

By insertions of the formulæ (2) and (3) in (4), we will obtain

$$R = A\,e\,(d + c) - A\,e\,(d - c'),$$

but it is evident that $c = c'$, therefore

$$R = A\,e\,2\,c, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

According to the force of gravity, the ascension c is such that if a body falls freely through the space c, it will obtain a velocity $= v$, which is equal to the velocity of the force P and the plane A, and known by the formula

$$c = \frac{v^2}{4\,g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which $g = 16.08$ feet, the space which a body falls in the first second, and $v =$ velocity in feet per second.

Example 3. **Fig. 2.**—The plane A moves at a velocity of 6 feet per second. What will the ascension c be?

$$c = \frac{6^2}{4 \times 16.08} = \frac{36}{67.32} = 0.56 \text{ feet.}$$

By insertion of the formula (6) in (5) we obtain

$$R = \frac{A\,e\,v^2}{2\,g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Example 4. **Fig. 2.**—What will be the resistance to a plane $A = 4.5$ square feet, moves at a velocity $v = 8$ feet per second, in fresh water $e = 62.5$ pounds?

$$R = \frac{4.5 \times 62.5 \times 8^2}{2 \times 16.08} = 560 \text{ pounds.}$$

This is a true calculation with neglected circumstances, namely: If there was no atmospheric pressure on the surface of the water, or if there were a number of tubes that would let down air to every part of the back side of the plane. As such is not the case, we must suit our formulæ and calculations to the circumstances, and find how the atmospheric pressure will affect them.

It is evident that, if the velocity is $v = o$, the pressure P and p will be as described in Fig. 1; but, if the velocity is $= v$, the pressure R and ascension c will still follow the same law as described in Fig. 2; but the pressure p will differ so that when the velocity reaches

$$v = \sqrt{4\ g\ (d + 32.92)}, \quad . \quad . \quad . \quad (8)$$

(in which d = depth, and 33.92 = the height of column of water which balances the atmosphere) the pressure p will be equal to o.

Example 5.—At a depth of 6 feet, what velocity is required to make the pressure $p = o$?

$$v = \sqrt{4 \times 16.08\ (6 + 32.92)} = 50 \text{ feet},$$

or about 34.4 miles per hour, a velocity not often exceeded by bodies in motion in fluid.

In the following formulæ, we will suppose that the velocity v does *not exceed* the formula (8), then we have the extremities of the pressure p at 1 and o, just as the velocity is o and v. Whatever the pressure p may be, it will balance a column of water less than the depth d, if the velocity is greater than o, or $v > o$. If the tube b be turned so that the aperture stands parallel with the motion and close to the back side of the plane, it is probable it would indicate a true descension c' for the pressure r, so that

$$c' : c = d : (d + 32.92),$$

of which

$$c' = \frac{c\ d}{d + 32.92}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Example 6. **Fig. 2.**—What will be the descension c', at a depth $d = 8$ feet, ascension $c = 6$ feet?

$$c' = \frac{6 \times 8}{8 + 32.92} = \frac{48}{40.92} = 1.175 \text{ feet.}$$

By insertion of the formula (9) in (3), we obtain

$$p = A\,e\left(d - \frac{c\,d}{d + 32.92}\right),$$

$$p = A\,e\,d\left(1 - \frac{c}{d + 32.92}\right), \quad . \quad . \quad (10)$$

From the formula (6) we have

$$c = \frac{v^2}{4\,g},$$

and

$$p = A\,e\,d\left(1 + \frac{v^2}{4\,g\,(d + 32.92)}\right), \quad . \quad . \quad (11)$$

$$P = A\,e\left(d + \frac{v^2}{4\,g}\right), \quad . \quad . \quad . \quad . \quad (12)$$

and

$$R = P - p,$$

we have

$$R = A\,e\left(d + \frac{v^2}{4\,g}\right) - A\,e\,d\left(1 + \frac{v^2}{4\,g\,(d + 32.92)}\right),$$

that is

$$R = \frac{A\,e\,v^2}{4\,g}\left(1 + \frac{d}{d + 32.92}\right), \quad . \quad . \quad (13)$$

This should be the true formula for calculating the resistance to planes in motion in fluid, but it is found that the resistance will be a little more owing to some air and friction in the water which diminishes the pressure p. Then, to make the formula durable in practical use, the term 32.92 must differ, and is found to be about 28.

In the accompanying table, the term

$$\delta = \frac{d}{d + 28}$$

is calculated at different values of d.

TABLE I.

Depth of the moving body below the surface of the water.	$\delta = \frac{d}{d + 28}$.			
	Feet.	3 inches.	6 inches.	9 inches.
0	0.0000	0.00885	0.01755	0.02603
1	0.0345	0.04275	0.0507	0.0587
2	0.0666	0.0744	0.0819	0.0894
3	0.0968	0.1041	0.1113	0.1184
4	0.1255	0.1321	0.1388	0.1460
5	0.1520	0.1583	0.1645	0.1760
6	0.1770	0.1830	0.1885	0.1945
7	0.2000	0.2057	0.2116	0.2170
8	0.2226	0.2280	0.2331	0.2384
9	0.2436	0.2486	0.2536	0.2585
10	0.2638	0.2683	0.2730	0.2774
11	0.2828	0.2870	0.2914	0.2361
12	0.3000	0.3041	0.3085	0.3130
13	0.3175	0.3215	0.3252	0.3290
14	0.3338	0.3373	0.3416	0.3452
15	0.3490	0.3530	0.357	0.3600
16	0.3645	0.3674	0.3713	0.3750
17	0.3780	0.3813	0.3850	0.3885
18	0.3922	0.3948	0.3982	0.4015
19	0.4050	0.4075	0.4110	0.4140
20	0.4175	0.4200	0.4230	0.426
21	0.429	0.432	0.435	0.438
22	0.441	0.443	0.446	0.449
23	0.452	0.454	0.457	0.460
24	0.462	0.464	0.467	0.470
25	0.473	0.475	0.477	0.479
26	0.482	0.485	0.488	0.490
27	0.492	0.495	0.497	0.499
28	0.501	0.503	0.506	0.508
29	0.510	0.512	0.514	0.516
30	0.5185	0.520	0.522	0.524
31	0.526	0.528	0.530	0.532
32	0.534	0.536	0.538	0.540
33	0.542	0.544	0.546	0.548
34	0.550	0.552	0.554	0.556
35	0.556	0.559	0.560	0.562
36	0.563	0.565	0.567	0.568

In the following formulæ we will substitute the quantities δ, instead of the term $\frac{d}{d + 28}$. Also, a co-efficient k instead of the symbols e and g, so that

$$k = \frac{e}{4\ g} = \begin{cases} 1 \text{ for salt water,} \\ 0.97 \text{ for fresh water,} \end{cases}$$

then, when the plane or body moves in salt water, the co-efficient k will not be seen, because then $k = 1$, and the formula for the resistance will be simply

$$R = A\ v^2\ k\ (1 + \delta), \quad . \quad . \quad . \quad (14)$$

Example 7. **Fig. 2.**

The plane . . .	$A = 7$ square feet
Moves at a velocity . .	$v = 16$ feet per second
At a depth of 6 feet .	$\delta = 0.231$
In fresh water . . .	$k = 0.97$
Require the resistance .	$R = ?$ in pounds.

$$R = 7 \times 16^2 \times 0.97\ (1 + 0.231) = 213.5 \text{ pounds.}$$

Example 8. **Fig. 2.**—What will be the resistance to the plane A when the dimensions are

Resistant area . . .	$A = 1$ square foot
Moves at a velocity . .	$v = 10$ feet per second
At a depth of 3 feet . .	$\delta = 0.0968$
In salt water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds.

$$R = 1 \times 10^2\ (1 + 0.0968) = 109.68 \text{ pounds.}$$

The same plane, moving at the same depth, with velocities

$v = 5$	the resistance	will be	$R = 27.3$	pounds.
$v = 2$	"	"	$R = 4.37$	"
$v = 1$	"	"	$R = 1.09$	"

Example 9. **Fig. 2.**—Dimensions of the plane being

Resistant area . . .	$A = 3$ square feet
Moves at a velocity . .	$v = 12$ feet per second
At a depth of 6 feet . .	$\delta = 0.177$
In fresh water . . .	$h = 0.97.$
Require the resistance .	$R = ?$ in pounds.

$R = 3 \times 12^2 \times 0.97\ (1 + 0.177) = 493$ pounds.

Fig. 3.—If the plane has an inclination to the direction of motion of an angle $= \Phi$ degrees, as shown in the figure, when not in motion, the pressure at right angle to the plane will be the same as described in Fig. 1, but the pressure in the direction of motion will be the total pressure multiplied by *sin.*ϕ, which is the same as the area projected parallel with motion, was the acting area for the resistance. In the following we will call

$A =$ area projected parallel with the motion.

$\Phi =$ angle of resistance *to* the motion.

$\phi =$ angle of incident *from* the motion.

The ascension and descension in the tubes a and b will also be as the *sines* for their corresponding angles of resistance and incident, so that the pressures P and p will be

$$P = A\,e\left(d + \frac{v^2\,sin.\Phi}{4\,g}\right), \quad . \quad . \quad . \quad (15)$$

$$p = A\,e\left(d - \frac{v^2\,sin.\phi\ \delta}{4\,g}\right), \quad . \quad . \quad . \quad (16)$$

When the two angles are $\Phi = \phi$, the formula for resistance will be

$$R = A\,k\,v^2\,sin.\Phi\,(1 + \delta), \quad . \quad . \quad . \quad (17)$$

Example 10. **Fig. 3.**—Suppose the plane has

An inclination of	. .	$\Phi = 42°$
Projected area	. . .	$A = 6.45$ square feet
Having a velocity	. .	$v = 5$ feet per second
At a depth of 8 feet	. .	$\delta = 0.222$ in the table
In fresh water	. . .	$k = 0.97$.

Require the resistance $R = ?$ in pounds.

$R = 6.45 \times 5^2 \times 0.97 \times sin.42° (1 + 0.222) = 127.7.$

Example 11. **Fig. 3.**—The plane has

An inclination of	. .	$\Phi = 30°$
Projected area	. . .	$A = 2.97$ square feet
Having a velocity	. .	$v = 6$ feet per second
At a depth of 6 feet	. .	$\delta = 0.177$ in the table
In salt water	. . .	$k = 1$

Require the resistance $R = ?$ in pounds.

$R = 2.97 \times 6^2\,sin.30°\,(1 + 0.177) = 62.7$ pounds.
$R = \qquad sin.20° \qquad = 42.8$ "
$R = \qquad sin.60° \qquad = 108.6$ "

Fig. 4.—If the plane has different angles of resistance and incident, the formula for resistance will be

$$R = A\,k\,v^2\,(sin.\Phi + sin.\phi\,\delta), \quad . \quad . \quad (18)$$

Example 12. **Fig. 4.**—The figure having dimensions of

Angle of resistance	. .	$\Phi = 43°$
Angle of incident	. .	$\phi = 35°$
Projected area	. . .	$A = 3.42$ square feet

Plate XXIV.

Fig. 4

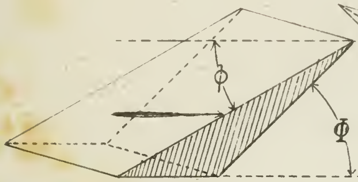


Fig. 5.

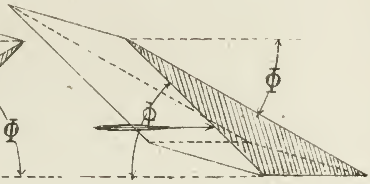


Fig. 6.

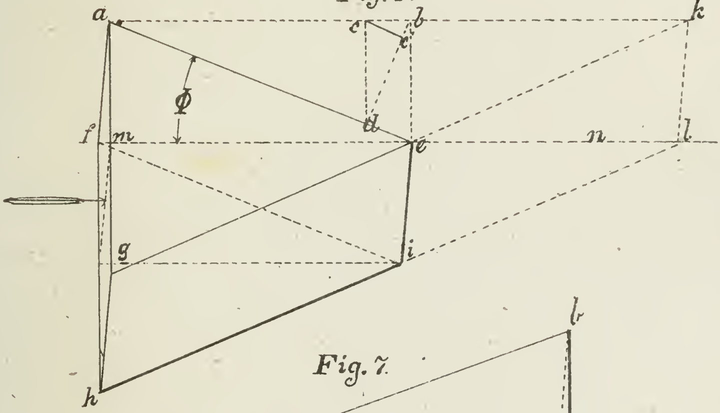
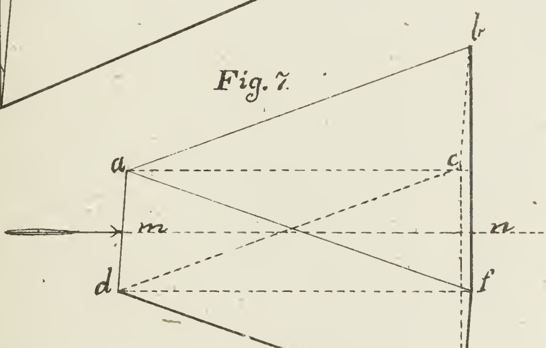
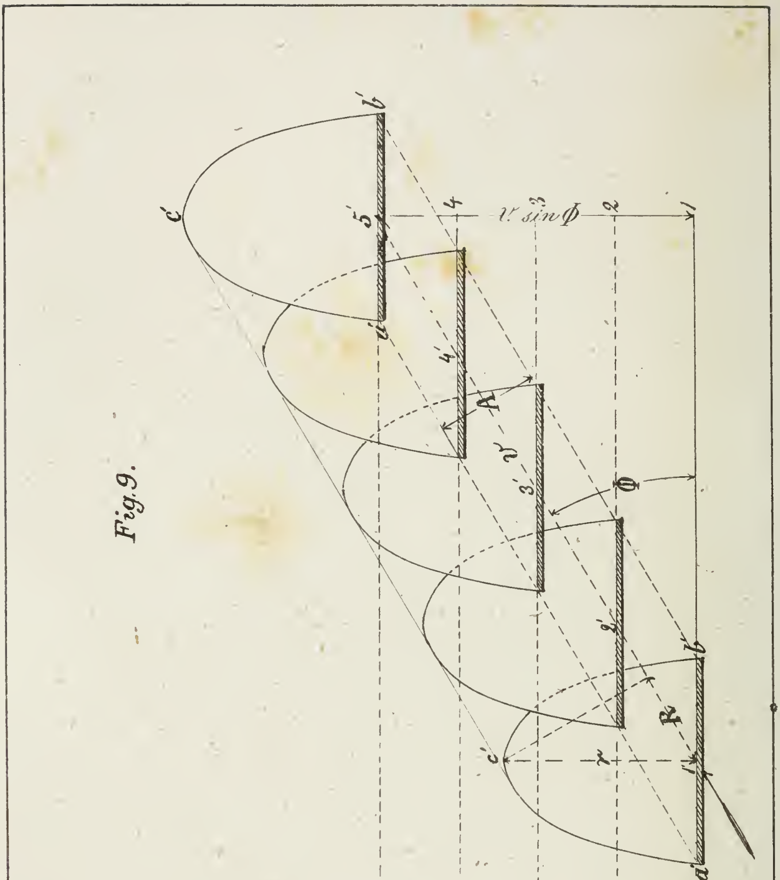


Fig. 7.



J.W. Nystrom.

Plate. XXV.



Moves at a velocity . . $v = 8$ feet per second
At a depth of 7 feet 3 inches $\delta = 0.206$, see table
In salt water . . . $k = 1$.
Require the resistance $R = ?$ in pounds.
$R = 3.42 \times 8^2\ (sin.43° + sin.35 \times 0.206) = 174.5$ pounds.

Example 13. **Fig. 5.**—The same plane as in Example 11, moves in the opposite direction at the same velocity and depth. Require the resistance $R = ?$ in pounds.

$R = 3.42 \times 8^2\ (sin.35 + sin.43 \times 0.206) = 155.8$.

Plate XXVI. **Fig. 8.**—Suppose the plane $a\ b$ moves in the direction of the arrow 1 2 3 4 and 5 (viewed from the top of the fluid), with a velocity = v feet per second, its resistance will be as described in Fig. 2. Let the mass of water that resists the plane be imagined by the inclusion of the curve $a\ c\ b$.

Fig. 9.—The plane $a'\ b'$ moves in the direction of the arrow 1′ 2′ 3′ 4′ and 5′ with a velocity so that it moves the space 1′ 5′ in the same time as the plane $a\ b$, Fig. 8, moves the space 1 5; then the velocities at right angle to the planes will be equal; consequently the resistance $a\ c\ b = a'\ c'\ b'$ in the directions at right angles to the planes.

The velocity of the plane $a\ b$ $\qquad 1\ 5 = V$,
" " $a'\ b'$ $\qquad 1'\ 5' = v$,

and

$$v : V = 1 : sin.\Phi.,$$

of which

$$V = v\ sin.\Phi., \quad . \quad . \quad . \quad . \quad . \quad (a)$$

If the plane

$$a\ b = a'\ b',$$

the resistances

$$a\ c\ b = a'\ c'\ b' = V^2\ (a\ b) = V^2\ (a'\ b'), \quad . \quad (b)$$

By the insertion of the formula (a) in (b) the resistance r, at right angle to the plane $a'\ b'$, will be

$$r = v^2\ sin.^2\Phi\ (a'\ b'), \quad . \quad . \quad . \quad . \quad (c)$$

but in the direction of the arrow 1′ 2′, the resistance R will be

$$r : R = 1 : sin.\Phi,$$

and

$$r = \frac{R}{sin.\Phi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

By the insertion of this value of r in the formula (c) we have

$$\frac{R}{sin.\Phi} = v^2\ sin.^2\Phi\ (a'\ b'),$$

and

$$R = v^2\ sin.^3\Phi\ sin.\Phi\ (a'\ b'), \quad . \quad . \quad . \quad (e)$$

that is, the resistance to the plane $a'\ b'$ should be in the proportion as the cube of the *sine* for the angle of inclination to the motion; but we have before determined to use the expression

$A =$ the area projected parallel to the motion;

then

$$A : a'\ b' = sin.\Phi : 1,$$

of which

$$A = a'\ b'\ sin.\Phi$$

inserted in the formula e will be

$$R = A\ v^2\ sin.^2\Phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

Which is the generally-received rule that resistance to bodies in motion in fluid, is as the square of the *sine* for the angle of resistance.

Let us again allude to the Figures 8 and 9. It will be found that the plane $a\ b$ (when in motion in the direction of the arrow 1 2) will come in contact with the same particles of water which before, gradually has obtained a motion; but the plane $a'\ b'$, in every successive moment, will come in contact with a *new* mass of water, whose inertia the plane must overcome by giving it the instant motion. (See **Fig. 9.**) The plane $a'\ b'$ moves from $1'$ to $2'$; when arrived at $2'$ it is in contact with the *new* mass of water, which moving mass extends far over the projected plane A. Let $m =$ the moving mass in Fig. 8, and $M =$ the moving mass in Fig. 9, their proportion will be

$$M : m = 1 : sin.\Phi,$$

and

$$R : R' = M : m,$$

of which

$$R = R'\ sin.\Phi,$$

inserted in the formula (f) will be

$$R\ sin.\Phi = A\ v^2\ sin.^2\Phi$$

and

$$R = A\ v^2\ sin.\Phi, \quad . \quad . \quad . \quad . \quad . \quad (g)$$

that is, the resistance should be in proportion direct as the *sine* of the angle of resistance.

Fig. 6.—The moving figure, having two inclined planes, $a\,e\,i\,f$ and $e\,g\,h\,i$, and a projected area $a\,g\,h\,f = A$,

and angle of resistance = Φ. The resistance to this figure will be the same as if it had only one inclined plane, with the same projected area and angle of resistance; that is, if the plane $a\ e\ i\ f$ was in the position $i\ e\ k\ l$, in the same line as the plane $e\ g\ h\ i$, and having the same projecting area $a\ g\ h\ f$. The resistance to a figure $a\ k\ l\ h\ f$ should be the same as to the figure $a\ e\ i\ h\ f$, and by the formula

$$R = A\ k\ v^2\ (sin.\Phi + \delta), \quad . \quad . \quad . \quad (19)$$

but it will be found that, when the figure moves the space $a\ b$, a particle of water at d will move the space $d\ b$, and, in the same direction as the moving figure, it will move the space $c\ b$. This velocity $c\ b$ will act in favor for the pressure p on the plane $a\ g\ h\ f = A$, but the quantity of water that goes over the corner a, with the velocity $c\ b$, will be measured by $cos.\Phi\ A$, and the velocity

$$c\ b = v\ sin.^2\Phi;$$

then the pressure p will be diminished by the weight of a column of water

$$W = A\ k\ v^2\ sin.^4\Phi\ cos.\Phi, \quad . \quad . \quad . \quad (20)$$

and the formula for resistance will be

$$R = A\ k\ v^2\ (sin.\Phi + sin.\phi\ \delta - sin.^4\Phi\ cos.\Phi), \quad (21)$$

Example 14. **Fig. 6.**—Dimensions of the figure.

Projected area . . .	$A = 4$ square feet
Angle of resistance . .	$\Phi = 18°$
Moves at a velocity . .	$v = 9.5$ feet per second
At a depth of 8.25 feet .	$\delta = 0.228$, Table I.
In fresh water . . .	$k = 0.97$

Require the resistance . $R = ?$ in pounds.

$R = 4 \times 0.97 \times 9.5^2 (sin.18° + 0.228 — sin.^4 18° \ cos.18°)$ $= 184.5$ pounds.

Example 15. **Fig. 6.**—When the dimensions are

Angle of resistance . . $\Phi = 30°$

Projected area . . . $A = 36$ square feet

Moves at a velocity . . $v = 15$ feet per second

At a depth of 10 feet . $\delta = 0.2638$

In salt water . . . $k = 1$

Require the resistance . $R = ?$ in pounds.

$R = 36 \times 15^2 (sin.30° + 0.2638 — sin.^4 30° \ cos.30°) =$ 5740 pounds.

Fig. 7.—The figure moves with its base forward in the direction $m\ n$. The formula for resistance will be

$$R = A\ k\ v^2\ (1 + sin.\phi\ \delta), \quad . \quad . \quad . \quad (22)$$

In this formula the terms are

$$sin.^4\Phi\ cos.\Phi = 0,$$

and

$$sin.\Phi = 1.$$

Example 16. **Fig. 7.**—The dimensions of the figure are

Angle of resistance . . $\Phi = 90°$

Angle of incident . . $\phi = 25°$

Projected area . . . $A = 12$ square feet

Moves at a velocity . . $v = 6$ feet per second

At a depth of 8 feet . . $\delta = 0.2226$

In salt water . . . $k = 1$

Require the resistance . $R = ?$ in pounds.

$R = 12 \times 6^2 (1 + sin.\ 25° \times 0.2226) = 472$ pounds.

Example 17. **Fig. 7.**—When the dimensions are

Angle of incidence . .	$\phi = 17°$
Projected area . . .	$A = 2.25$ square feet
Having a velocity . .	$v = 12$ feet per second
At a depth of 7 feet . .	$\delta = 0.2$
In fresh water . . .	$k = 0.97$
Require the resistance .	$R = ?$ in pounds.

$R = 2.25 \times 0.97 \times 12^2 \,(1 + sin.\, 17° \times 0.2) = 332.5.$

Fig. 10.—When the two angles of resistance and incident are equal, the formula for resistance will be

$$R = A\, k\, v^2\, sin.\Phi \;(1 + \delta - sin.^3\Phi\; cos.\Phi). \qquad (23)$$

Example 18. **Fig. 10.**—Dimensions are

Angles of resistance and incident	$\Phi = 16°$
Projected area . . .	$A = 1.75$ square feet
Velocity	$v = 11$ feet per second
At a depth of 10 feet .	$\delta = 0.2638$
In salt water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds.

$R = 1.75 \times 11^2 sin.16\; (1 + 0.2638 - sin.^3 16°\; cos.16°) =$ 72.6 pounds.

Example 19. **Fig. 11.**—The figure having different angles of resistance and incident.

Angle of resistance . .	$\Phi = 15°$
Angle of incident . .	$\phi = 12°$
Projected area . . .	$A = 3$ square feet
At a depth of 6 feet . .	$\delta = 0.177$
At a velocity . . .	$v = 12$ feet per second
In salt water . . .	$k = 1$

Plate XXVI.

Fig. 10

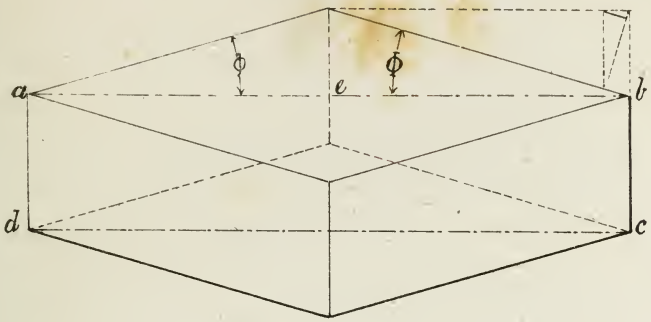


Fig. 11.

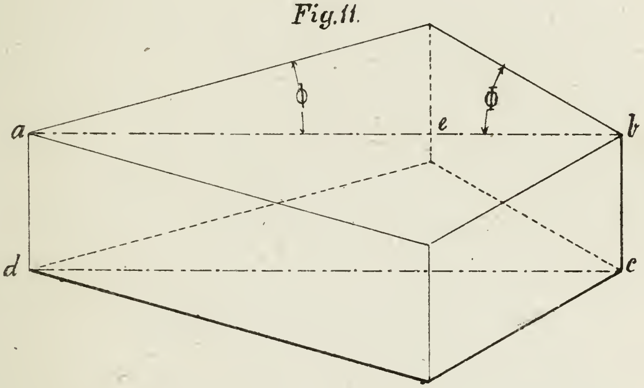
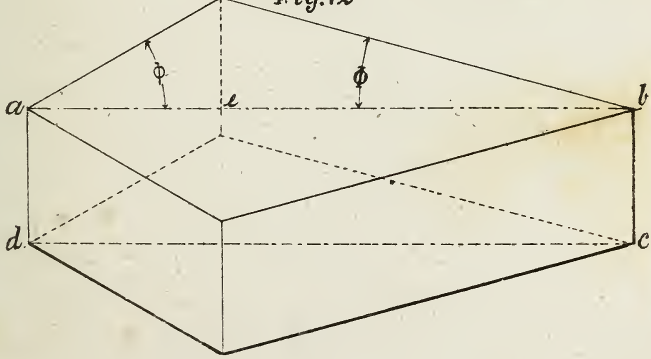


Fig. 12



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Require the resistance . $R = ?$ in pounds.

$R = 3 \times 12^2 (sin.15 + sin.12 \times 0.177 — sin.^4 15 \times cos.15)$
$= 125.6.$

Example 20. **Fig. 12.**—The figure having the same dimensions as Fig. 11, and moves at the same depth and velocity as in the Example 19.

Require the resistance $R = ?$ in pounds.

$R = 3 \times 12^2\ (sin.12 + sin.15 \times 0.177 — sin.^4 12° \times cos.12°)$
$= 108.5$ pounds.

In these examples (19 and 20) the results should have been greater in the latter, according to results from experiments and generally-received rules among ship-builders. When, the friction being added, there will be less difference between their actual resistance, but it will not be less for Fig. 11 than for Fig. 12.

Friction and Cohesion.

Fig. 15.—A plane a (or body) in motion in fluid, parallel to itself, has two essential resistances to sustain, namely:—

Friction between the surface and the fluid, which is proportional to the hydrostatic pressure or depth δ.

Second. *Cohesion* between the surface and the fluid, which is constant at any depth d, but depends on the nature of the surface of the plane. If this surface is very smooth, or polished and greasy, the *cohesion* is very slight, or $= o$, and sometimes negative; but when the surface is not greasy or polished, but rough, and has been in the water for some time, so that slime is collected on the same, the *cohesion* is greatest. But, whatever this *cohesion* may be, it is measured by the altitude of a column of the fluid in which the plane is immersed, so that if $c =$ altitude of the column which balances the cohesion, and $\delta =$ altitude of the column that balances the hydrostatic pressure, it is evident that $\delta + c =$ altitude of a column which balances the total friction on the immersed plane.

Now, if the plane moves at a velocity $= v$ feet per second, the particles of water which are near or in contact with the plane will also move with a velocity proportional to

$$\frac{v}{s}(\delta + c), \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

in which $\frac{1}{s} =$ a *fraction* expressing the roughness of the surface of the plane.

Figs. 13 and **14** represent a section of a small portion of the plane, on a large scale, so that *a a a* are projections on the surface of the plane at the distances *b b b*. It is evident that those projections will act each as a resistant plane to the direction of motion, and each of them having an angle of resistance of which

$$\mathbf{a}\, sin.\phi = \frac{1}{s} = \quad . \quad . \quad . \quad . \quad . \quad (25)$$

is a measure of the roughness of the plane, and the area **a** of the whole plane is a measure of the number

Plate XXVII.

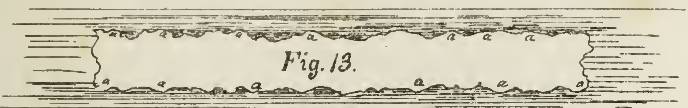


Fig. 13.

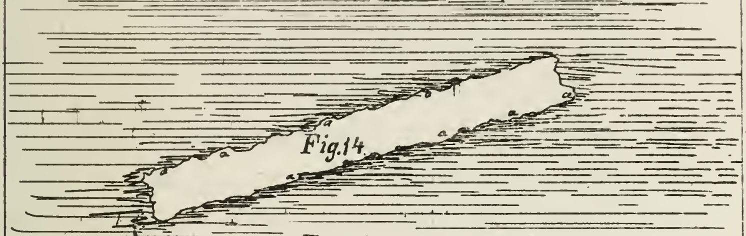


Fig. 14.

Fig. 15.

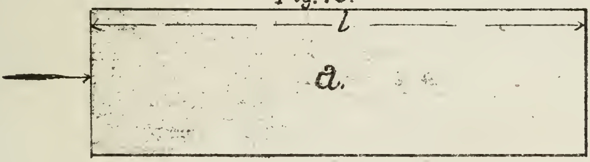


Fig. 16.

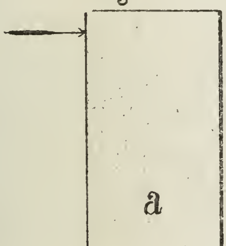


Fig. 17.

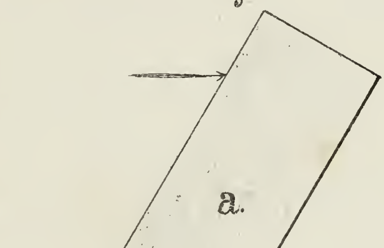
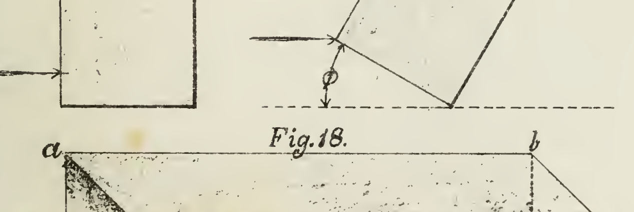


Fig. 18.



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of all those roughnesses $a\ a\ a$, or $\frac{1}{s}$; but the number of roughnesses in the direction of motion comes in contact with the same particles of water (fluid) which have obtained a velocity by the foregoing roughness, and do not act in full as resistant on the latter ones; therefore, the square root must be extracted from the number of roughnesses in the direction of motion; so that the total resistance for the plane, or what in this case is called the *friction*, should be measured by the formula

$$f = \frac{\mathbf{a}\ v^2}{s\ \sqrt{l}}\,(\delta + c), \quad . \quad . \quad . \quad . \quad (26)$$

or, by insertion of the symbols e and g, we have

$$f = \frac{\mathbf{a}\ e\ v^2}{\sqrt{l}\ s\ 4\ g}\,(\delta + c), \quad . \quad . \quad . \quad (27)$$

To ascertain the two quantities s and c only from theory is rather a difficult matter; we will, therefore, allude to experiments and approximate those values only for water, which is the principal fluid for which this is intended, and is found to be about

$$s = 580,$$

when the surface of friction is wood smooth planed, and been in the water for some time, and slime has collected on the same. For the same kind of friction surface, the *cohesion* is about

$$c = 2 \text{ feet},$$

the altitude of a column of water that would balance the *cohesion*. Then f will be expressed in pounds by the formula

$$f = \frac{\mathbf{a}\, v^2}{580\, \sqrt{l}}\,(\delta + 2), \quad . \quad . \quad . \quad . \quad (28)$$

which formula gives the friction as near as may be desirable for practical purposes.

$\mathbf{a}$ = friction area, which is parallel to the motion, dead area.

Fig. 14.—The plane having an inclination *to* and *from* the direction of motion. When the plane has an inclination *to* the direction of motion, the angle of resistance of the roughness will increase as the angle of inclination; but, when the inclination is *from* the direction of motion, the angle of resistance of the roughness will be diminished, say $f = 1$ when the friction plane is parallel to the motion. Then

$$f = 1 + \sin.^2\Phi \cos.\Phi$$

friction on the plane inclined *to* the direction of motion, and

$$f = 1 - \sin.^2\Phi \cos.\Phi$$

will be the friction on the plane inclined *from* the direction of motion.

Example 21. **Fig. 7.**—Suppose the plane $\mathbf{a}$ to be 12 feet 6 inches long, and breadth 1 foot 9 inches, the edges being so sharp that no resistance to them exists.

Friction area $\mathbf{a} = 2 \times 12.5 \times 1.75 = 43.7$ square feet

Moves at a depth . . $d = 5.5$ feet

Co-efficient . . . $\delta = 0.164$

With a velocity . . $v = 7.66$ feet per second
Require the friction . $f = ?$ in pounds.

$$f = \frac{43.7 \times 7.66^2}{580 \sqrt{12.5}} (0.164 + 2) = 0.27 \text{ pounds.}$$

Example 22. **Fig. 16.**—What will be the friction on a board of the same dimensions as in Example 21, but moves in the direction of its breadth, as represented by Fig. 16?

$$f = \frac{43.7 \times 7.66^2}{580 \sqrt{1.75}} (0.164 + 2) = 0.72 \text{ pounds.}$$

Example 23. **Fig. 17.**—What will be the friction on a board moving in a direction as represented by Fig. 17. Dimensions:—

Length . . . . $= 10$ feet 3 inches
Breadth . . . . $= 1$ foot 8 inches
Friction area $a = 10.25 \times 1.666 \times 2 = 34$ square feet
Moves at a velocity . . $v = 9$ feet per second
At a depth . . . . $d = 10$ feet
Co-efficient . . . . $\delta = 0.264$
The angle being . . . $\phi = 38°$
Require the friction . . $f = ?$ in pounds.

$$f = \frac{34 \times 9^2 (0.264 + 2)}{580 \sqrt{1.666 \times sec.38°}} = 7.4 \text{ pounds.}$$

Example 24. **Fig. 6.**—To find the friction on the dead areas of the two triangles $a\,e\,g$ and $f\,i\,h$, the figure having the same dimensions as in the Example 14, and moves at the same depth and velocity.

$a\,g = 2.675$ feet
$m\,e = 4.15$ "

Friction area $\mathbf{a} = 4.15 \times 2.675 = 11.1$ square feet.

The length for friction on a triangle will be only half its length in the direction of motion, or

$$l = \frac{m\ e}{2} = \frac{4.15}{2} = 2.075 \text{ feet.}$$

Require the friction $f = ?$ in pounds.

$$f = \frac{11.1 \times 9.5^2}{580\sqrt{2.075}}(0.228 + 2) = 2.68 \text{ pounds.}$$

Fig. 6.—When the friction area has an inclination *to* the direction of motion, as represented by Fig. 6, the formula for its friction will be

$$f = \frac{a\, v^2\, (1 + sin.^2\Phi\ cos.\Phi)\ (2 + \delta)}{580\sqrt{l}}, \quad . \quad (30)$$

in which $\mathbf{a}$ = area of the whole inclined plane multiplied by $cos.\Phi$, the angle of inclination to the direction of motion, which, in this Fig. 6 will be

$$\text{a} = cos.\Phi\ (a\ e\ i\ f + e\ g\ h\ i) = e\ i\ o\ m = m\ e \times e\ i.$$

Example 25. **Fig. 6.**—To find the friction on the inclined planes $a\ e\ i\ f$ and $e\ g\ h\ i$. The figure has the same dimensions as in Example 14, which will be

The length . . $l = m\ e = 4.15$ feet

The height . . . $e\ i = 1.5$ feet

Friction area $\mathbf{a} = m\ e \times e\ i \times 2 = 4.15 \times 1.5 \times 2 = 12.45$ square feet

At a depth of 8.25 feet . $\delta = 0.228$

Moves at a velocity . . $v = 9.5$ feet per second

Angle of resistance . . $\Phi = 38°$

Require the friction . . $f = ?$ in pounds.

$$f = \frac{12.45 \times 9.5^2 \times (1 + sin.^2 38 \times cos. 38)(2 + 0.228)}{580 \sqrt{4.15}} =$$

2.75 pounds.

Then the actual resistance to the Fig. 6 will be the sum of the resistance R, and the two frictions on the dead and inclined friction areas, from

Example 14 resistance		$R = 184$	pounds
"	24 friction	$f = 2.68$	"
"	25 "	$f = 2.75$	"
Actual resistance		$R = 189.93$	pounds.

Fig. 7.—The inclined friction area having its inclination *from* the direction of motion, the formula for its friction will be

$$f = \frac{\mathbf{a}' v^2 (1 - sin.^2 \phi \; cos. \phi)(2 + \delta)}{580 \sqrt{l}}, \quad . \quad (31)$$

Example 26. **Fig. 7.**—To find the friction on the planes $a\ b\ c\ d$ and $a\ d\ e\ f$, the figure moves in the direction $m\ n$, shown by the arrow. The dimensions of the figure to be the same as in the Example 16, viz:—

Angle of incident . .	$\phi = 25°$
The length $m\ n$ . . .	$l = 4.29$ feet
The height	$ad = 3$ feet
Friction area $\mathbf{a}' = 4.29 \times 3 \times 2$	$= 25.74$ square feet
At a depth of 8 feet . .	$\delta = 0.222$
Velocity	$v = 6$ feet per second
Require the friction . .	$f = ?$ in pounds.

10

$$f = \frac{25.74 \times 6^2 (1 - sin.^2 25^\circ \times cos.25)\,(2 + 0.222)}{580\,\sqrt{4.29}} = 1.43.$$

Fig. 10.—The moving figure having friction areas *to* and *from* the direction of motion. The formula for friction on both the inclined planes will be the sum of the two formulæ (30) and (31), which will appear in one formula, as

$$f = \frac{v^2\,(2 + \delta)}{580\,\sqrt{l}} \Big[\mathbf{a}\ (1 + sin.^2\Phi\ cos.\Phi)$$

$$\mathbf{a} + (1 - sin.^2\phi\ cos.\phi) \Big], \quad . \quad . \quad . \quad (32)$$

in which

l = length of the whole figure in feet,

$\mathbf{a}$ = friction area *to* the direction of motion,

$\mathbf{a}'$ = friction area *from* the direction of motion.

It will *not* be correct to calculate the friction from the two formulæ (30) and (31) separately and add them together, owing to the quantity l. To simplify the calculations and setting up of the formula (32), it will be best to separate the two terms within the parenthesis as follows:—

$$q = \mathbf{a}\ (1 + sin.^2\Phi\ cos.\Phi), \quad . \quad . \quad . \quad (33)$$

$$q' = \mathbf{a}'\ (1 - sin.^2\phi\ cos.\phi), \quad . \quad . \quad . \quad (34)$$

$$f = \frac{v^2\ (2 + \delta)(q + q')}{580\,\sqrt{l}}, \quad . \quad . \quad . \quad (35)$$

Example 27. **Fig. 11.**—What will be the frictions on the inclined planes, the figure having the same dimensions as in the Example 19?

The height . . . $da = 0.75$ feet

$ae = 1.87$ feet

$eb = 2.35$ feet

Length . $l = 1.87 + 2.35 = 4.22$ feet

Friction area $\mathbf{a}' = 2.35 \times 0.75 \times 2 = 3.52$ square feet

Friction area $\mathbf{a} = 1.87 \times 0.75 \times 2 = 2.28$ square feet

Angle of resistance . . $\Phi = 15°$

Angle of incident . . $\phi = 12°$

Moves at a velocity . . $v = 12$ feet per second

At a depth of 6 feet . . $\delta = 0.177$

Require the friction . . $f = ?$ in pounds.

$$q = 2.28\,(1 + \sin.^2 15° \times \cos.15°) = 2.425,$$

$$q' = 3.52\,(1 - \sin.^2 12° \cos.12°) = 3.38,$$

$$f = \frac{12^2\,(2 + 0.177)\,(2.425 + 3.37)}{580\,\sqrt{4.22}} = 1.52 \text{ pounds.}$$

Example 28. **Fig. 12.**—The figure having the same dimensions as in the preceding example but moves in an opposite direction at the same depth and velocity.

Friction area . . . $\mathbf{a} = 3.52$ square feet

Friction area . . . $\mathbf{a}' = 2.28$ square feet

Angle of resistance . . $\Phi = 12°$

Angle of incident . . $\phi = 15°$

Require the friction . . $f = ?$ in pounds.

$$q = 3.52\,(1 + \sin.^2 12° \times \cos.12°) = 3.67,$$

$$q' = 2.28\,(1 - \sin.^2 15° \times \cos.15°) = 2.13,$$

$$f = \frac{12^2\,(2 + 0.177)\,(3.67 + 2.13)}{580\,\sqrt{4.22}} = 1.52 \text{ pounds.}$$

Fig. 10.—When the two angles of resistance and incident are equal, the friction areas **a** and **a**′ will

also be equal, and the formula for friction will be simply the formula (28), and the friction area

$$\mathbf{a} = 2 \times a\, b\, c\, d.$$

Fig. 18.—A parallelepiped in motion, parallel with its length, should have the same resistance as a plane with the same projecting area, omitting the friction; but it is found by experiments that sometimes the friction acts in favor of the resistance, and at other times it increases the same. There is apparently a certain proportion of areas for friction and resistance most favorable for the total resistance, owing to the friction and cohesion causing a motion of the water, which increases the pressure p; then R will be *diminished.* But, when the resistant area A is very small, the pressure p will also be small, and the friction and cohesion remains the same, it is evident that R will be *increased.* On the opposite, if A is large, and the friction area $\mathbf{a}$ is very small, the effect on the pressure p will be trifling.

Then, if it is *only* the motion of the water, caused by the friction, that diminishes the resistance R, it is probable that the minums of R will be, when the formula (25) is,

$$f = \frac{\mathbf{a}\, v^2\, (2 + \delta)}{580\, \sqrt{l}} = A\, v^2\, \delta, \text{ the pressure } p,$$

which is indicated by the tube b, Fig. 2. Then the proper proportion of A and $\mathbf{a}$ will be

$$\mathbf{a} : 580\, \sqrt{l} = A\, \delta : (2 + \delta),$$

or

$$\mathbf{a} : A = 580\ \delta \sqrt{l} : (2 + \delta),$$

and

$$\frac{\mathbf{a}}{A} = \frac{580\ \delta \sqrt{l}}{(2 + \delta)}.$$

$l = a\ b$, length of the parallelepiped,

$s = \sqrt{A}$, suppose the base A to be a square;

then, $\mathbf{a} = 4\ s\ l$, and $A = s^2$, we have

$$\frac{4\ s\ l'}{s^2} = \frac{580\ \delta \sqrt{l'}}{(2 + \delta)},$$

$$\frac{4\ s\ l'}{s^2 \sqrt{l'}} = \frac{4 \sqrt{l'}}{s} = \frac{580\ \delta}{(2 + \delta)}.$$

$$\frac{\sqrt{l'}}{s} = \frac{580\ \delta}{4\ (2 + \delta)} = \frac{145\ \delta}{(2 + \delta)}.$$

Then the proper proportion of l and s should be

$$\sqrt{l'} : s = 145\ \delta : (2 + \delta).$$

By taking different values of δ we obtain the corresponding proper proportions of l and s, which will be, when $s = 1$,

$$\sqrt{l'} = \frac{s\ 145\ \delta}{(2 + \delta)},$$

$$l' = \frac{21225\ \delta^2}{(2 + \delta)^2}, \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Example 29. **Fig. 18.**—What will be the proper length of the parallelepiped, in order to give the least possible resistance when moving at a depth of 6 feet, the co-efficient is $\delta = 0.177$?

$$l' = \frac{21225 \times 0.177^2}{(2 + 0.177)^2} = 140.$$

This length, 140, is about 16 or 18 times what actually occurs in practice, consequently there must be another motion of the water, originated at the *fore-end* of the moving body, which diminishes the pressure p, and it is probable that the profit of this motion is proportional to the length l, until the limit of the proper length l', so that this latter motion, together with the friction, will be equal to the pressure p at the proper length l'. This is the very point which makes it difficult to set up theory for bodies in motion in fluid; but, by a few experiments expressly for that purpose, it would be easily settled. It is this motion which causes the vessel to have less resistance when the greatest immerse section $\mathfrak{N}$ is more forward.

At present, there is no other means but to approximate the value of this latter motion, in connection with the friction, which, for practical purposes, will suffice to calculate the proper length l' from the formula

$$l' = \left(\frac{145\,d}{d^2 + 30\,d + 50}\right)^2, \quad . \quad . \quad . \quad . \quad (b)$$

From this formula the proper proportions of s and l' are calculated at different depths d, also the proper proportions of $\mathbf{a}$ and A.

TABLE II.

s : l'	Depth in feet. d	A : a'
1 : 2.845	1	1 : 11.40
1 : 5.83	2	1 : 23.33
1 : 7.83	3	1 : 31.31
1 : 9.07	4	1 : 36.2
1 : 9.85	5	1 : 39.3
1 : 10.16	6	1 : 40.6
1 : 10.34	7	1 : 41.3
1 : 10.35	8	1 : 41.4
1 : 10.24	9	1 : 41.0
1 : 10.10	10	1 : 40.3
1 : 9.63	12	1 : 38.5
1 : 8.84	15	1 : 35.4
1 : 7.56	20	1 : 30.2
1 : 6.40	25	1 : 25.5
1 : 5.46	30	1 : 21.8
1 : 4.13	40	1 : 16.1

At a depth of about 8 feet, the parallelepiped should be longest in proportion to its side s, and have the least possible resistance. At depths of more or less than 8 feet, it should be shorter when the co-efficient for the friction is 580. If the surface of the parallelepiped is smoother, it will be longer; and shorter if more rough.

Letters will denote:—

l' = proper length in proportion to s.

l = any other length more or less than l'.

R = resistance calculated from the formula (13).

f' = friction and motion calculated from the formula (d).

R = actual resistance including friction.

Then, when $l < l,'$ the actual resistance should be

$$R = R - f', \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

When $l > l'$, the actual resistance will be

$$R = v^2 k \left[A + \frac{(\delta + 2)}{180} \left(\frac{\mathbf{a}}{\sqrt{l}} - \frac{'\mathbf{a}}{\sqrt{l'}} \right) \right], \quad . \qquad (37)$$

$$f' = \frac{\mathbf{a}\, v^2}{180 \sqrt{l}} (2 + \delta), \quad . \quad . \quad . \quad . \quad . \qquad (d)$$

Example 30. **Fig. 18.**—What will be the actual resistance to this figure at the four different lengths?

Example 1.	$l = 5$	feet	$< l'$
" 2.	$l = 10$	"	$= l'$ nearly
" 3.	$l = 15$	"	$> l'$
" 4.	$l = 20$	"	$> l'$

The figure to be moved in salt water at

A velocity . . .	$v = 10$ feet per second
A depth	$d = 6$ feet
Its co-efficient . . .	$\delta = 0.177$
The side	$s = 1$ foot square
Resistant area . . .	$A = 1$ square foot.

Ex. 1. Friction area $\mathbf{a} = 4 \times 5 = 20$ square feet.

$$R = 1 \times 10^2 (1 + 0.177) = 117.7 \text{ pounds.}$$

$$f = \frac{20 \times 10^2}{180 \ \sqrt{5}} (0.177 + 2) = 10.85 \text{ pounds.}$$

$$R = 117.7 - 10.85 = 106.95 \text{ pounds.}$$

Ex. 2. Friction area $\mathbf{a} = 4 \times 10 = 40$ square feet.

$$R = 117.7, \text{ same as Ex. 1.}$$

$$f' = \frac{40 \times 10^2}{180 \sqrt{10}} (0.177 + 2) = 15.35 \text{ pounds.}$$

$$R = 117.7 - 15.35 = 102.35 \text{ pounds.}$$

Ex. 3. Friction area $\mathbf{a} = 4 \times 15 = 60$ square feet
Proper friction area $'\mathbf{a} = 4 \times 10.16 = 40.64$ square feet
Proper length $l' = 10.16$ feet.

Require the actual resistance from the formula (36) ?

$$R = 10^2\left[1 + \frac{(0.177 + 2)}{180}\left(\frac{60}{\sqrt{15}} - \frac{40.64}{\sqrt{10.16}}\right)\right] = 103.32.$$

Ex. 4. Friction area . . . $\mathbf{a} = 4 \times 20 = 80$
Proper area . . . . $\mathbf{a}' = 40.64$
Proper length . . . . $l' = 10.16$
Require the actual resistance R ?

$$R = 10^2\left[1 + \frac{(0.177 + 2)}{180}\left(\frac{80}{\sqrt{20}} - \frac{40.64}{\sqrt{10.16}}\right)\right] = 106.16.$$

Example 31.—What will be the actual resistance to a parallelepiped of dimensions

Length . . . . $l = 12$ feet
Breadth . . . . $b = 1.5$ feet
Thick . . . . . $t = 0.75$ feet
Resistance area $A = 1.5 \times 0.75 = 1.125$ square feet

$$b\,l = 2 \times 1.5 \times 12 = 36$$
$$t\,l = 2 \times 0.75 \times 12 = 18$$

Friction area . . . $\mathbf{a} = 54$ square feet
Move at a velocity . . $v = 9$ feet per second
At a depth . . . . $d = 8$ feet
In salt water . . . $k = 1.$

$$\frac{\mathbf{a}}{A} = \frac{54}{1.125} = 48.$$

Proper area at 8 feet $\mathbf{a}' = 41.4 \times 1.125 = 46.6 < 48,$

consequently, the actual resistance to be calculated from the formula (37),

$$\text{Proper length } l' = \frac{10.35\ (1.5 + 0.75)}{2} = 11.65.$$

Require the actual resistance = ? in pounds.

$$R = 9^2\left[1.125 + \frac{(0.177+2)}{180}\left(\frac{54}{\sqrt{12}} - \frac{46.6}{\sqrt{11.65}}\right)\right]$$

= 95.6 pounds.

Example 32.—What will be the resistance to a cylinder with dimensions

Length . . . . $l = 17.5$ feet

Diameter . . . . $D = 1.33$ feet

Resistant area $A = 1.33^2 \times 0.785 = 1.39$ square feet

Friction area $\mathbf{a} = 1.33 \times 3.14 \times 17.5 = 73.1$ square feet

Moves at a velocity . . $v = 12$ feet per second

At a depth . . . $d = 4$ feet

Proper area at 4 feet $\mathbf{a}' = 36.2 \times 1.39 = 50.3$ square feet

$$\frac{\mathbf{a}}{A} = \frac{73.1}{1.39} = 52.6 > 50.3,$$

$$\text{Proper length } l' = \frac{50.3}{3.14 \times 1.33} = 12 \text{ feet}$$

In fresh water . . . $k = 0.97$.

Require the actual resistance R = ? in pounds.

$$R = 12^2 \times 0.97\left[1.39 + \frac{(0.177+2)}{180}\left(\frac{73.1}{\sqrt{17.5}} - \frac{50.3}{\sqrt{12}}\right)\right]$$

= 200 pounds.

Fig. 17. Plate XXVIII.—A cube moving in a direction as shown by the arrow, with one of its edges foremost, its angle of resistance is evidently

$$\Phi = 45^\circ.$$

Example 33. **Fig. 17.**—Suppose the side of the cube to be 2 feet, we have the

Area of resistance	$A = 2\sqrt{2^2+2^2} = 5.656$ square feet
Angle of resistance . .	$\Phi = 45° = \phi$
Moves at a velocity . .	$v = 8$ feet per second
At a depth . . .	$d = 6$ feet
The co-efficient . . .	$\delta = 0.177$
In salt water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds,

from the formula (23).

$$R = 5.656 \times 8^2 \times sin.45\ (1 + 0.177 - sin.^3 45 \times cos.45)$$
$$= 237 \text{ pounds.}$$

Fig. 18.—A cube moving in a direction parallel with two opposite corners, its angle of resistance will be

$$\Phi = 35°.$$

Example 34. **Fig. 18.**—The cube having the same dimensions as in the Example 26, the resistant area will be

$$A = 7.75 \text{ square feet.}$$

Require the resistance $R = ?$ in pounds.

$$R = 7.75 \times 8^2 \times sin.35\ (1 + 0.177 - sin.^3 35 \times cos.35)$$
$$= 290 \text{ pounds.}$$

Figures with curved lines.—The figures of which we before treated are straight lined; when curved, there will be a more complicated calculation for their resistance, owing to the curves having an infinite number of angles of resistances, which must be united to one *mean angle of resistance.* By reference to tables, calculated for all the different curves, as *circle, ellipse, parabola*, and *hyperbola*, &c., their resistance can be calculated simply by the formula (14), in which the term $(1 + \delta)$ would be two co-efficients from the tables. To set up those tables, it would be well to have experiments expressly for them, although they can be calculated without; but, in this book, too much of the subject is omitted to furnish them all; we will furnish only one for the circle, calculated for every degree to 90°, which may enable to approximate the angle of resistance of a vessel.

Fig. 19.—When the moving figure is a circle or a part thereof, the mean angle of resistance will be found in the Table III., so calculated that, when the centre for the circle arc $a\ b$ lays on the line y (the centre line of the figure which is parallel with the motion), its corresponding angle of resistance will be found in the column $\Phi\ y$, and the column w is the circle arc $a\ b$ in degrees. When the centre for the circle arc $a\ b$ is on the line x, its corresponding angle of resistance will be found in the column $\Phi\ x$.

In the figures 19, 20, and 21, the circle arc $a\ b$ is 90°, and its centre on both the lines x and y. Opposite 90°, in the column $w°$, will be found 39° 34′, in the columns $\Phi\ x$ and $\Phi\ y$, which is the mean angle of resistance for a circle.

Plate XXVIII

Fig. 17.

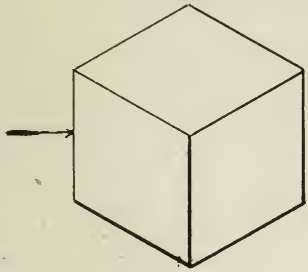


Fig. 18.

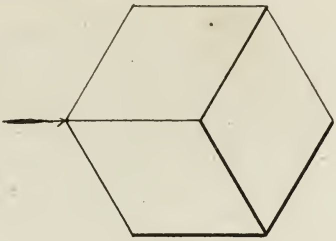


Fig. 19.

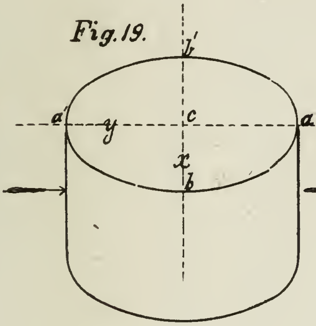


Fig. 20.

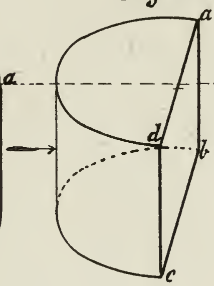


Fig. 21.

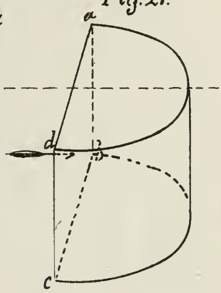


Fig. 22.

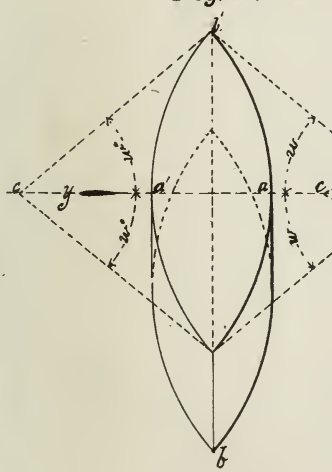
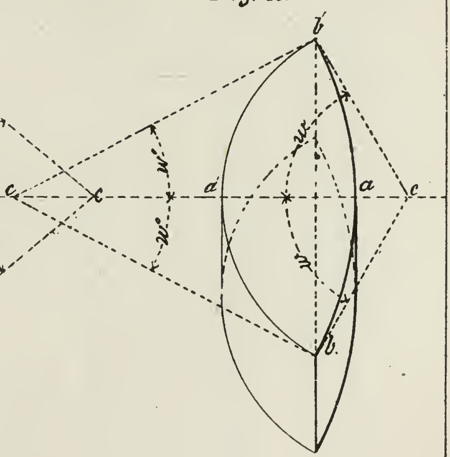


Fig. 23.



J. W. Nystrom.

TABLE III.

$x\ \Phi$	$w°$	$y\ \Phi$	$x\ \Phi$	$w°$	$y\ \Phi$
0° 58′	**1**	89° 6′	22° 48′	**46**	63° 36′
1 28	**2**	88 12	23 16	**47**	63 02
1 58	**3**	87 41	23 43	**48**	62 28
2 28	**4**	87 09	24 12	**49**	61 54
2 58	**5**	86 40	24 38	**50**	61 10
3 27	**6**	86 08	25 4	**51**	60 36
3 58	**7**	85 33	25 32	**52**	60 02
4 27	**8**	85 00	25 57	**53**	59 28
4 57	**9**	84 27	26 24	**54**	58 54
5 26	**10**	83 44	26 50	**55**	58 20
5 57	**11**	83 10	27 16	**56**	57 48
6 26	**12**	82 36	27 42	**57**	57 14
6 57	**13**	82 02	28 58	**58**	56 40
7 26	**14**	81 88	28 34	**59**	56 16
7 57	**15**	80 54	28 59	**60**	55 31
8 26	**16**	80 20	29 25	**61**	54 57
8 57	**17**	79 46	29 50	**62**	54 23
9 26	**18**	79 12	30 15	**63**	53 49
9 56	**19**	78 38	30 40	**64**	53 16
10 25	**20**	78 03	31 4	**65**	52 45
10 55	**21**	77 29	31 28	**66**	52 12
11 25	**22**	76 55	31 52	**67**	51 39
11 54	**23**	76 21	32 15	**68**	51 06
12 23	**24**	75 47	32 32	**69**	50 32
12 53	**25**	75 13	33 02	**70**	49 58
13 22	**26**	74 41	33 25	**71**	49 25
13 51	**27**	74 06	33 49	**72**	48 52
14 20	**28**	73 35	44 13	**73**	48 19
14 49	**29**	72 00	34 34	**74**	47 46
15 18	**30**	72 25	34 57	**75**	47 12
15 46	**31**	71 52	35 18	**76**	46 40
16 16	**32**	71 19	35 40	**77**	46 08
16 45	**33**	70 47	36 00	**78**	45 36
17 14	**34**	70 15	36 20	**79**	45 04
17 42	**35**	69 43	36 40	**80**	44 32
18 11	**36**	69 11	37 00	**81**	44 00
18 40	**37**	68 37	37 19	**82**	43 29
19 8	**38**	68 17	37 38	**83**	42 58
19 35	**39**	67 35	37 55	**84**	42 28
20 3	**40**	66 58	38 12	**85**	41 85
20 29	**41**	66 25	38 29	**86**	41 28
20 58	**42**	66 52	38 46	**87**	40 59
21 26	**43**	65 23	39 06	**88**	40 30
21 53	**44**	64 44	39 20	**89**	40 02
22 20	**45**	64 10	39 34	**90**	39 34

Fig. 19.—A cylinder moving in a direction parallel to its bases (which are supposed to be at right angle to its axis), angle of resistance is only

$$\Phi = 39° \, 34'.$$

Example 35. **Fig. 19.**—Suppose the cylinder to have the length equal to the side of the cube in the Examples 33 and 34, and a diameter equal to circumscribe a square of the cube, then its resistant area will be the same as in the Example 33,

Which is	$A = 5.656$ square feet
Angle of resistance . .	$\Phi = 39° \, 34'$
Moves at a velocity . .	$v = 8$ feet per second
At a depth of 6 feet . .	$\delta = 0.177$
Require the resistance . .	$R = ?$ in pounds.

$R = 5.656 \times 8^2 \times sin.39° \, 34'(1 + 0.177 - sin.^3 39° \, 34' \times cos.39° \, 34') = 225.3$ pounds.

Figs. 20 and **21.**—The cylinder being cut in two equal parts, parallel through its axis, the one part to be moved with the square side forward, and the other with its hemisphere forwards.

Example 36. **Fig. 20.**—The half-cylinder to have the same dimensions as the cylinder in the preceding example, moving at the same depth and velocity in the direction of the arrow.

Require the resistance $R = ?$

$R = 5.656 \times 8^2 \, (1 + sin. \, 39° 34' \times 0.177) = 401$ pounds.

Example 37. **Fig. 21.**—Require the resistance when moving with the hemisphere forward, and having the same dimensions as before.

$R = 5.656 \times 8^2$ ($sin.39° \ 34' + 0.177 - sin.^4 39° \ 34 \times cos.39° \ 34'$) $= 248.5$ pounds.

Fig. 22.—The centre c for the arc $a\ b$ being on the line y, and its angle $w° = a\ c\ b'$, its angle of resistance in the column $y\ \Phi$.

Example 38. **Fig. 22.**—Suppose the

Angle of the arc is . .	$w° = 40°$
Then angle of resistance .	$\Phi = 66° \ 58'$, Table III.
Area of resistance . .	$A = 4$ square feet
Moves at a velocity .	$v = 7$ feet per second
At a depth of 5 feet .	$\delta = 0.152$, Table I.
In salt water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds.

$R = 4 \times 7^2 \times sin.66° \ 58'$ ($1 + 0.152 - sin.^3 66° \ 58' \times cos.66° \ 58'$) $= 153$ pounds.

Fig. 23.—The centres for the arcs $a\ b$ and $a'\ b'$ being both on the line y, but $w°$ is different for resistance and incident, then the resistance will be calculated from the formula (21), and angles of resistance and incident in the column $y\ \Phi$.

Example 39. **Fig. 23.**—Suppose the figure moves in the direction shown by the arrow, and the

Angle $w° = 43°$ resistance .	$\Phi = 65° \ 23'$, Table III.
Angle $w = 32°$ incident .	$\phi = 71° \ 19'$, "
Area of resistance . .	$A = 5$ square feet
Moves at a velocity . .	$v = 10$ feet per second
At a depth of 6 feet 6 in. .	$\delta = 0.1885$
In salt water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds.

$$R = 5 \times 10^2 \,(sin.65° \; 23' + sin.71° \; 19' \times 0.1885 - sin.^4 65° \; 23' \times cos.65° \; 23') = 401.5 \text{ pounds.}$$

Example 40. **Fig. 23.**—The figure to be moved in an opposite direction of the arrow, and having the same dimensions as in the Example 39, and moves at the same depth and velocity, then

Angle of resistance . . .	$\Phi = 71° \; 19'$
Angle of incident . . .	$\phi = 65° \; 23'$
Require the resistance . .	$R = ?$ in pounds.

$$R = 5 \times 10^2 \,(sin.71° \; 19' + sin.65° \; 23' \times 0.1885 - sin.^4 71° \; 19' \times cos.71° \; 19') = 430 \text{ pounds.}$$

Fig. 24.—The centres c for the arc $a\,b$ being on the line x, and its angle $w = a\,c\,b$, its angle of resistance in the column $x\,\Phi$.

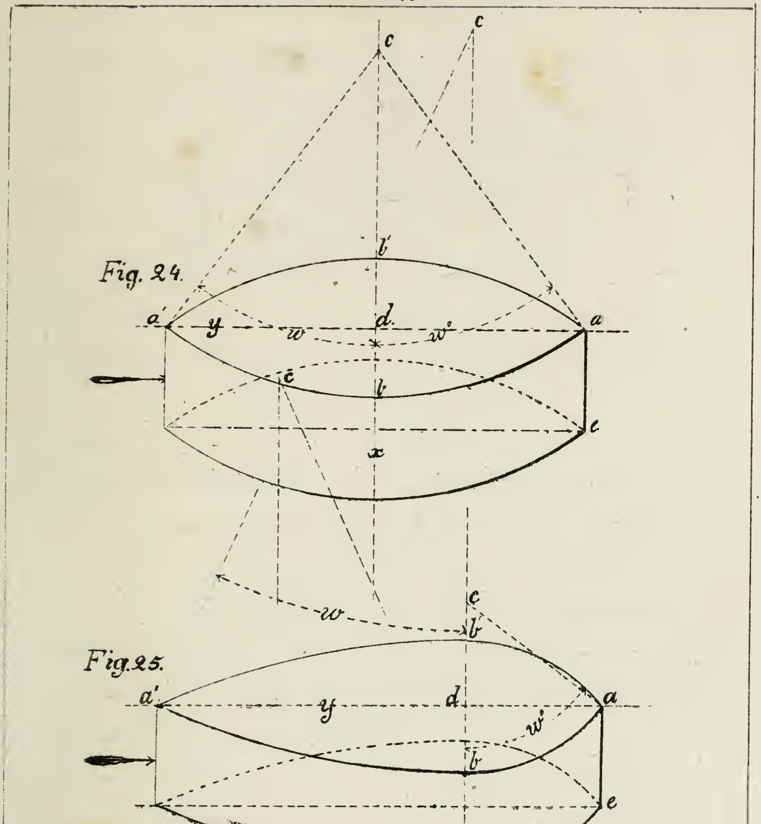
Example 41. **Fig. 24.**—The figure to be moved in the direction of the arrow with dimensions—

The angle of the arc . .	$w = 28°$
Angle of resistance and incident	$\Phi = 14° \; 20'$
Resistant area . . .	$A = 14$ square feet
Moves at a velocity . .	$v = 12$ feet per second
At a depth of 6 feet . .	$\delta = 0.177$
In fresh water . . .	$k = 0.97.$
Require the resistance .	$R = ?$ in pounds, from the formula (23).

$$R = 14 \times 12^2 \times 0.97 \times sin.14° \; 20' \,(1 + 0.177 - sin.^3 14° \; 20' \times cos.14° \; 20') = 559 \text{ pounds.}$$

Fig. 25.—The centres for the arcs $a\,b$ and $a\,b'$ being both on the line x, but different angles $w°$. The

Plate XXIX.



angle of resistance and incident to be found in the column $x\ \Phi$. The resistance from the formula (21).

Example 42. **Fig. 25.**—The figure moves in the direction of the arrow with dimensions—

Angle $w° = 35°$ resistance .	$\Phi = 17° 42'$
Angle $w = 28°$ incident .	$\phi = 14° 20'$
Resistance area . . .	$A = 10$ square feet
Moves at a velocity . .	$v = 15$ feet per second
At a depth of 6 feet . .	$\delta = 0.177$
In fresh water . . .	$k = 1$
Require the resistance .	$R = ?$ in pounds.

$$R = 10 \times 15^2 (sin.17° 42' + sin.14° 20' \times 0.177 - sin.^4 17° 42' \times cos.17° 42') = 763 \text{ pounds.}$$

Example 43. **Fig. 26.**—The figure moves in an opposite direction to the arrow, and having the same dimensions as in the Example 42, and moves at the same depth and velocity, then

Angle of resistance . . .	$\Phi = 14° 20'$
Angle of incident . . .	$\phi = 17° 42'$
Require the resistance . .	$R = ?$ in pounds.

$$R = 10 \times 15^2 (sin.14° 20' + sin.17° 42' \times 0.177 - sin.^4 14° 20' \times cos.14° 20') = 669 \text{ pounds.}$$

The length $a\ a'$ can be found by the angle w and breadth $b\ b'$.

$a\ a' = l$ length of the figure,

$b\ b' = b$ breadth of the figure,

$a\ e = h$ height of the figure,

$a\ c = r$ radii of the circle arc.

Then

$$A = b\,h,$$

and

$$r = \frac{l^2 + b^2}{4\,b}, \quad . \quad . \quad . \quad . \quad . \quad (38)$$

$$sin.w = \frac{l}{2\,r} = \frac{2\,l\,b}{l^2 + b^2}, \quad . \quad . \quad . \quad (39)$$

By solving this formula in favor of l, we have

$$sin.w\ (l^2 + b^2) = 2\,l\,b,$$

$$sin.w\ l^2 + sin.w\ b^2 = 2\,l\,b,$$

$$sin.w\ l^2 - 2\,l\,b = -\,sin.w\ b^2,$$

$$l^2 - \frac{2\,l\,b}{sin.w} = -\,b^2,$$

$$l^2 - \frac{2\,l\,b}{sin.w} + \frac{b^2}{sin.^2 w} = \frac{l^2}{sin.^2 w} - b^2,$$

$$\left(l - \frac{b}{sin.w}\right)^2 = b^2\left(\frac{1}{sin.^2 w} - 1\right),$$

$$l - \frac{b}{sin.w} = b\sqrt{\frac{1}{sin.^2 w} - 1},$$

lastly

$$l = b\left(\frac{1}{sin.w} + \sqrt{\frac{1}{sin.^2 w} - 1}\right), \quad . \quad . \quad (40)$$

Example 44. **Fig. 25.**—Suppose

The angle of the curve $a\,b$ is . . $w^\circ = 35^\circ$

And the breadth $b\,b'$ . . . $b = 4$ feet

Require the length . . . . $a\,a' = ?$ in feet

$$d\,a = \frac{4}{2}\left(\frac{1}{sin.35^\circ} + \sqrt{\frac{1}{sin.^2 35^\circ} - 1}\right) = 6.34 \text{ feet},$$

The angle of the arc $b\,a'$ . . . $w = 28^\circ$

Breadth $b\,b'$ . . . . . $b = 4$ feet,

$$d\,a' = \frac{4}{2}\left(\frac{1}{sin.28^\circ} + \sqrt{\frac{1}{sin.^2 28^\circ} - 1}\right) = 8 \text{ feet.}$$

The whole length $a\,a' = 6.34 + 8 = 14.34$ feet.

Example 45. **Fig. 25.**—The figure is to have the same dimensions as in the preceding examples. What will be the friction on the curved sides.

The height $a\,e = \frac{A}{b} = \frac{10}{4} = 2.5$ feet.

Length . . . . $l = 14.34$

Friction area $\mathbf{a} = 2 \times 6.34 \times 2.5 = 31.7$ square feet

Friction area $\mathbf{a}' = 2 \times 8 \times 2.5 = 40$ square feet

Angle of resistance . . $\Phi = 17^\circ\ 42'$

Angle of incident . . $\phi = 14^\circ\ 20'$

Moves at a velocity . . $v = 15$ feet per second

At a depth . . . $d = 6$ feet

Co-efficient . . . $\delta = 0.177$

Require the friction . . $f = ?$ in pounds.

$$q = 31.7\ (1 + sin.^2 17^\circ\ 42' \times cos.17^\circ\ 42') = 34.4,$$

$$q' = 40\ (1 - sin.^2 14^\circ\ 20' \times cos.14^\circ\ 20') = 37.6,$$

$$f = \frac{15^2\ (2 + 0.177)\ (34.4 + 37.6)}{580\ \sqrt{14.34}} = 16 \text{ pounds.}$$

Example 46. **Fig. 26.**—The figure having the same dimensions as in the preceding example, but move in an opposite direction.

Friction area . . . $\mathbf{a} = 40$ square feet

Friction area . . . $\mathbf{a}' = 31.7$ square feet

Angle of resistance . . $\Phi = 14^\circ\ 20'$

Angle of incident . . $\phi = 17^\circ\ 42'$

Require the friction . . $f = ?$ in pounds.

$$q = 40\ (1 + \sin.^2 14^\circ\ 20' \times \cos. 14^\circ\ 20') = 42.3,$$

$$q' = 31.7\ (1 - \sin.^2 17^\circ\ 42' \times \cos. 17^\circ\ 42') = 28.9,$$

$$f = \frac{15^2\ (1 + 0.177)\ (42.3 + 28.9)}{580\ \sqrt{14.34}} = 15.9.$$

$$\text{Fig. 25} \begin{cases} \text{Example 42} & R = 763 \\ \text{Example 45} & f = 16 \\ \text{Actual resistance} & R = 779 \text{ pounds.} \end{cases}$$

$$\text{Fig. 26} \begin{cases} \text{Example 43} & R = 669 \\ \text{Example 46} & f = 15.9 \\ \text{Actual resistance} & R = 684.9 \text{ pounds.} \end{cases}$$

Fig. 27.—A force R being applied between two planes A and $\mathfrak{A}$ immersed in water at a depth d. It is evident the planes will obtain velocities so that their resistance will be equal to the applied force R, or

$v =$ velocity of the plane A,

$M =$ velocity of the plane $\mathfrak{A}$.

We have

$$R = A\,k\,v^2\,(1 + \delta) = \mathfrak{A}\,k\,M^2\,(1 + \delta), \quad . \quad (41)$$

and

$$v = \sqrt{\frac{R}{A\,k\,(1 + \delta)}} = M\sqrt{\frac{\mathfrak{A}}{A}}, \quad . \quad . \quad (42)$$

$$M = \sqrt{\frac{R}{\mathfrak{A}\,k\,(1 + \delta)}} = v\sqrt{\frac{A}{\mathfrak{A}}}, \quad . \quad . \quad (43)$$

Example 47. **Fig. 27.**—Dimensions of the planes are

$A = 6$ square feet

$\mathfrak{A} = 11$ square feet

The force applied . . . . $R = 483$ pounds

At a depth in salt water . . $d = 6$ feet

Co-efficient . . . . $\delta = 0.177$

Require the velocities . . $v = ?$ and $M = ?$

$$v = \sqrt{\frac{483}{6\ (1 + 0.177)}} = 8.3 \text{ feet per second,}$$

$$M = 8.3 \sqrt{\frac{6}{11}} = 6.1 \text{ feet per second.}$$

Fig. 28.—A force R being applied between the plane A and the figure $\aleph$, whose angles of resistance $= \Phi$, incident $= \phi$. We have

$$R = A\,k\,v^2\,(1 + \delta) = \aleph\,k\,M^2\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\,cos.\Phi), \quad . \quad . \quad . \quad . \quad . \qquad (44)$$

Suppose the velocities $M + v = 1$. $v = S$, which in the first part of this book is termed *slip*. Then

$$M = (1 - S),$$

and

$$S = (1 - M),$$

$$R = A\,S^2\,(1 + \delta) = \aleph\,(1 - S)^2\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\,cos.\Phi), \quad . \quad . \quad . \quad . \quad . \qquad (45)$$

and

$$S\sqrt{A(1+\delta)} = (1-S)\sqrt{\aleph(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi cos.\Phi)},$$

$$S\sqrt{A\,(1+\delta)} = \sqrt{\aleph\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\,cos.\Phi)} - S\sqrt{\aleph\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\,cos.\Phi)},$$

$$S = \frac{\sqrt{\aleph\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\,cos.\Phi)}}{\sqrt{A\,(1+\delta)} + \sqrt{\aleph\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi cos.\Phi)}}, \qquad (46)$$

For convenience, in setting up this formula and calculating, it will be better to substitute other characters for the quantities under the root marks; for instance—

$$\oplus = \sqrt{\mathfrak{N}\,(sin.\Phi + sin.\phi\,\delta - sin.^4\Phi\; cos.\Phi)}, \qquad (47)$$

$$\bigcirc = \sqrt{A\,(1 + \delta)}, \quad . \quad . \quad . \quad . \quad . \qquad (48)$$

Then the formula for the slip will appear as

$$S = \frac{\oplus}{\bigcirc + \oplus}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

Example 48. **Fig. 28.**—The figures having dimensions—

Projected area . . .	$\mathfrak{N} = 39$ square feet
Area of the plane . . .	$A = 18$ square feet
Angle of resistance . .	$\Phi = 21°$
Angle of incident . . .	$\phi = 18°$
Moves at a depth . . .	$d = 6$ feet
Co-efficient	$\delta = 0.177$
Require the slip . . .	$S = ?$ per cent.

$$\oplus = \sqrt{39\,(sin.21 + sin.18 \times 0.177 - sin.^4 21 \times cos.21)}$$
$= 3.93,$

$$\bigcirc = \sqrt{18\,(1 + 0.177)} = 4.52,$$

$$S = \frac{3.93}{4.57 + 3.93} = 0.462, \text{ or } 46.2 \text{ per cent.}$$

Fig. 28.—Suppose the figure to have the angle of resistance $\Phi = \phi$ incident, and the curved side $a'\,b\,a$ being an arc of a circle with a radii

$$r = \frac{l^2}{4\,b'} + \frac{b'}{4}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (50)$$

Suppose the figure to be immersed so that the plane $a\,b\,a'\,b'$ is level with the surface of the water, and $\mathfrak{d} =$ draft of water,

$d =$ depth of the centre of gravity of $\mathfrak{A}$ under the surface of the water.

In this figure, the projected area $\mathfrak{A}$ is a parallelogram, and the draft of water

$$\mathfrak{d} = \frac{\mathfrak{A}}{b},$$

and

$$d = \frac{\mathfrak{d}}{2}.$$

Another imaginary breadth for the angle w° is

$$b' = \frac{2\, d\, b}{\mathfrak{d}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

This value of b is to be inserted in the formula (50).

$$r = \frac{l^2\, \mathfrak{d}}{8\, d\, b} + \frac{d\, b}{2\, \mathfrak{d}}, \quad . \quad . \quad . \quad . \quad (51)$$

The area of the load-line $a\ b\ a'\ b'$, will be found by the formula

$$a = \frac{\pi\, r^2\, w}{90} - l\left(r - \frac{b'}{2}\right), \quad . \quad . \quad . \quad (52)$$

This, multiplied by the draft of water, will be the cubic contents of the figure; but, before inserting the correct draft of water, we must prepare it to suit our next coming irregular figures, and will be

$$\mathfrak{d}' = \frac{2\, d\, \mathfrak{A}}{\mathfrak{d}\, b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

This, multiplied by the formula (52), will be the cubic contents of the figure

$$C = \frac{2\, d\, \mathfrak{A}\, \pi\, r^2\, w}{90\, \mathfrak{d}\, b} - \frac{2\, d\, \mathfrak{A}\, l}{\mathfrak{d}}\left(r - \frac{b'}{2}\right), \quad . \quad (53)$$

By solving this formula in favor of w°, we obtain the angle of resistance from the cubic contents, and the greatest immerse section, and

$$\frac{2\,d\,\text{☒}\,\pi\,r^2\,w}{90\,\mathfrak{d}\,b} = C + \frac{2\,d\,\text{☒}\,l}{\mathfrak{d}\,b}\left(r - \frac{b'}{2}\right),$$

$$2\,d\,\text{☒}\,\pi\,r^2\,w^\circ = C\,90\,\mathfrak{d}\,b + 90 \times 2\,d\,\text{☒}\left(r - \frac{b'}{2}\right),$$

$$w^\circ = \frac{C\,90\,\mathfrak{d}\,b}{2\,d\,\text{☒}\,\pi\,r^2} + \frac{90\,l}{\pi\,r^2}\left(r - \frac{b'}{2}\right),$$

$$w = \frac{90}{\pi\,r^2}\left[\frac{C\,\mathfrak{d}\,b}{2\,d\,\text{☒}} + l\left(r - \frac{b'}{2}\right)\right], \quad . \quad . \quad . \qquad (54)$$

This is the formula by which the angle w° for a vessel should be calculated, with the supposition that the cubic contents of the displacement should be near the formula (53). As such cannot always be the case, we must make the formula more dependent on the displacement.

Formula (54).—Multiply the terms by the factor

$$\frac{90\,b}{\pi\,r^2},$$

we have

$$w = \frac{90\ C\,\mathfrak{d}\,b}{2\,\pi\,r^2\,d\,\text{☒}} + \frac{90\,l}{\pi\,r} - \frac{90\,d\,b\,l}{\pi\,r^2\,\mathfrak{d}},$$

of which

$$w = \frac{90\,b}{\pi\,r^2}\left(\frac{C\,\mathfrak{d}}{2\,d\,\text{☒}} - \frac{d\,l}{\mathfrak{d}}\right) + \frac{90\,l}{\pi\,r}.$$

The first term of this formula is only a small fraction of the angle w°, which, without detriment, can be taken out, and the remainder will be

$$w = \frac{90\,l}{\pi\,r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (c)$$

Plate XXX.

Fig. 27.

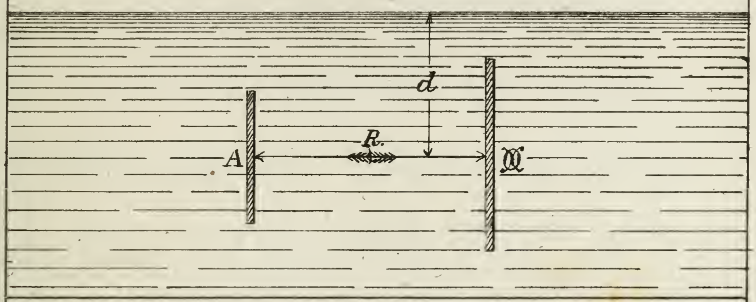


Fig. 28.

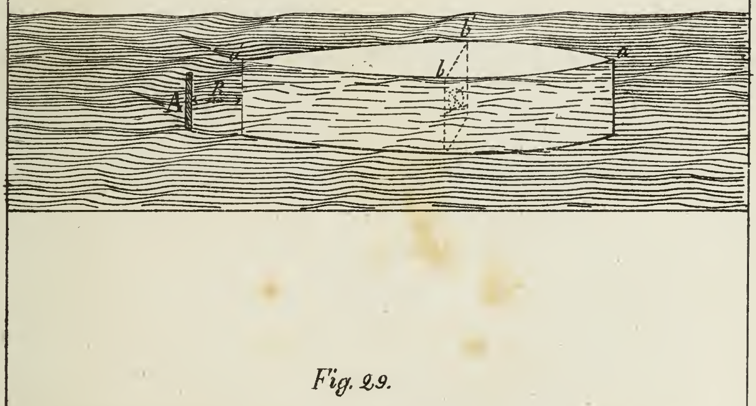
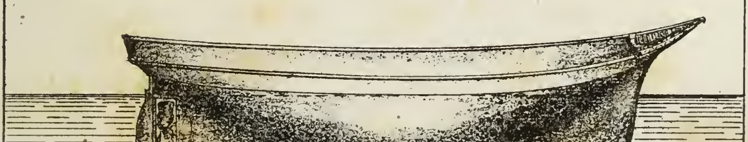


Fig. 29.



Here the length l must be dependent on the displacement exceeding the formula (53). In the common reasonable proportions of steamboats, the displacement is generally in the neighborhood of the formula (53); then the angle w will be very near

$$w = \frac{200\ C d}{\mathfrak{N}\ \mathfrak{d}\ r}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (55)$$

In which are contained the following three suppositions:—

1st. The vessel to have *no hollow* lines.

2d. The greatest immerse section to be *in or near the middle of the length l.*

3d. The lines to be *about equal fore and aft.*

If the displacement of the vessel is given in tons (of 2240 pounds) multiply the

$$\text{tons} = Q \text{ by } \left\{ \begin{array}{l} 34.9 \text{ for salt water} \\ 35.9 \text{ for fresh water} \end{array} \right\} = C \text{ displace-}$$

ment in cubic feet.

The formulæ and calculations heretofore treated are based on that

Resistance to bodies in motion in fluid is direct as the sine for the angle of resistance. This gives the resistance but little more than actually occurs in practice. When the figures, or resistant-planes, are of a curved or round form, at right angle to the motion, as a cone moving with its vertex forward, the water being thrown out in every direction, which again diminishes the resistance nearly as the *sine* for the angle of inclination, or resistance to a cone will be nearer as the *square of the sine for the angle of inclination.* A well-constructed vessel can be compared with a cone, as it is expressly suited to the motion of the water. Then the formulæ (44), (45), (46), and (47) will be—

$$R = A\,k\,v^2\,(1+\delta) = \mathfrak{X}\,k\,M^2\,\sin.^2\Phi\,(1+\delta - \sin.^2\Phi\,\cos.\Phi), \tag{56}$$

$$R = A\,S^2(1+\delta) = \mathfrak{X}\,(1-S)\,\sin.^2\Phi\,(1+\delta - \sin.^2\Phi\,\cos.\Phi), \tag{57}$$

$$S = \frac{\sin.\Phi\,\sqrt{\mathfrak{X}\,(1+\delta - \sin.^2\Phi\,\cos.\Phi)}}{\sqrt{A\,(1+\delta)} + \sin.\Phi\,\sqrt{\mathfrak{X}\,(1+\delta - \sin.^2\Phi\,\cos.\Phi}}, \tag{58}$$

and

$$\oplus = \sin.\Phi\,\sqrt{\mathfrak{X}\,(1+\delta - \sin.^2\Phi\,\cos.\Phi)}, \quad . \tag{59}$$

To this formula (59) must be added the friction. We have before supposed the vessel to have the angles of resistance and incident about equal, then the friction areas **a** and **a**′ will also be equal, and the formula (32) will appear as

$$f = \frac{2\,\mathbf{a}\,v^2}{580\,\sqrt{l}}\left(\delta + 2 + \frac{v^2\,\sin.^3\Phi\,\cos.\Phi}{4\,g}\right), \tag{60}$$

The term within the parentheses,

$$\frac{v^2\,\sin.^3\Phi\,\cos.\Phi}{4\,g},$$

is only a small fraction of the friction f, which, without detriment, can be relinquished, and the friction

$$f = \frac{2\,\mathbf{a}\,v^2}{580\,\sqrt{l}}\,(\delta + 2), \quad . \quad . \quad . \quad . \tag{28}$$

in which the area 2 **a** should be calculated from

$$2\,\mathbf{a} = \frac{l}{2\,\mathfrak{d}}\,(\mathfrak{N} + 4\,\mathfrak{d}^2), \quad . \quad . \quad . \quad . \quad (61)$$

This value of $2\,\mathbf{a}$ inserted in the formula (28) will be

$$f = \frac{v^2 \sqrt{l}\,(\delta + 2)\,(\mathfrak{N} + 4\,\mathfrak{d}^2)}{1160\,\mathfrak{d}}, \quad . \quad . \quad (62)$$

which should be the friction of the vessel in pounds, and

$$\oplus = \sqrt{\frac{\sqrt{l}\,(\delta + 2)\,(\mathfrak{N} + 4\,\mathfrak{d}^2)}{1160\,\mathfrak{d}}}, \quad . \quad . \quad (63)$$

is the value which is to be added to the formula (59) before it is inserted in the formula (49).

$$\bigcirc = \sqrt{A\,(1 + \delta)}, \quad . \quad . \quad . \quad . \quad (48)$$

$$S = \frac{\oplus}{\bigcirc + \oplus}, \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Example 49. **Fig. 29.**—This is a vessel for which the angle of resistance is to be calculated. Her dimensions are

Tonnage of displacement .	$Q = 882$ tons
Capacity	$C = 30782$ cubic feet
Length in the load line .	$l = 180$ feet
Beam	$b = 26$ feet
Draft of water	$\mathfrak{d} = 13$ feet
Depth of centre of gravity .	$d = 5.75$ feet
Greatest immerse section .	$\mathfrak{N} = 300$ square feet
Require the angle of resistance	$\Phi = ?$

$$r = \frac{180^2 \times 13}{8 \times 5.75 \times 26} + \frac{5.75 \times 26}{2 \times 13} = 357.25 \text{ feet,}$$

$$w = \frac{200 \times 30782 \times 5.75}{300 \times 13 \times 357.25} = 25.4^\circ.$$

See column . . .	$x\ \Phi$	and w	Table III.
From	13° 22′	" **26**	
Subtract . . .	12° 53′	" **25**	

Multiply . . 0.4 × 0° 29′ = 11.6′
To . . . . 12° 53′
Add . . . . 0° 11′

Angle of resistance Φ = 13° 4′ the answer.

Example 50. **Fig. 29.**—The same vessel to have a propeller of dimensions,

Diameter . . . . . $D = 11$ feet
Pitch . . . . . $P = 27$ feet
Require the acting area . $A = ?$ in square feet.

From the formula (81), page 80.

$$A = \frac{2.5 \times 11^3}{\sqrt{27^2 + 3.14^2 \times 11^2}} = 113.4 \text{ square feet.}$$

Example 51. **Fig. 29.**—The vessel to have the same dimensions as in the example 49.

The Vessel.

Depth of centre of gravity . $d = 5.75$ feet
Co-efficient . . . . $\delta = 0.17$
Greatest immerse section . $\mathfrak{B} = 300$ square feet
Draft of water . . . $\mathfrak{d} = 13$ feet
Length in the load-line . . $l = 180$ feet
Require the two values . . $\oplus = ?$

$$\oplus = sin.13° 4' \sqrt{300\,(1 + 0.17 - sin.^2 13° 4' \times cos.13° 4')}$$
$= 4.13$,

$$\oplus = \sqrt{\frac{\sqrt{180}\,(0.17 + 2)\,(300 + 4 \times 13^2)}{1160 \times 13}} = 1.37,$$

$$\oplus = 4.13 + 1.37 = 5.5.$$

The Propeller.

Depth of centre of gravity .	$d = 6.5$ feet
Co-efficient	$\delta = 0.188$ feet
Acting area . . .	$A = 113.4$ square feet
Require the value . .	$\bigcirc = ?$

$$\bigcirc = \sqrt{113.4\,(1 + 0.188)} = 11.56,$$

Require the slip $S = ?$

$$S = \frac{5.5}{11.56 + 5.5} = 0.32 \text{ or } 32 \text{ per cent.}$$

Figs. 30, 31, and **32.**—These are three different vessels, supposed to have equal length, beam, and draft of water, and propellers of equal diameter. In the following examples, we will calculate the angle of resistance for the vessels and slip of their propellers.

Example 52. **Fig. 30.**—Dimensions of the vessel being,

Length in the load-line . .	$l = 100$ feet
Beam	$b = 20$ feet
Draft of water . . .	$\mathfrak{d} = 10$ feet
Depth of centre of gravity .	$d = 2.8$ feet
Greater immerse section .	$\mathfrak{X} = 100$ square feet
Tonnage of displacement .	$Q = 143$ tons
Capacity	$C = 5000$ cubic feet
Require the angle of resistance	$\Phi = ?$

$$r = \frac{100^2 \times 10}{8 + 2.8 \times 20} + \frac{2.8 \times 20}{2 \times 10} = 226 \text{ feet,}$$

$$w = \frac{200 \times 5000 \times 2.8}{100 \times 10 \times 226} = 12.4,$$

See column . . .	$x\ \Phi$	and w	Table III.
From	6° 57′	" **13°**	
Subtract . . .	6° 26′	" **12°**	
Multiply . . 0.4 ×	0° 29′ = 11.6′		
To	6° 25′		
Add	0° 55′.6		

Angle of resistance Φ = 6° 55′.6 the answer.

Example 53. **Fig. 30.**—Dimensions of the propeller being

Diameter . . . . . $D = 8.33$ feet

Pitch . . . . . . $P = 21.5$ feet

Require the acting area . $A = ?$ in square feet.

$$A = \frac{2.5 \times 8.33^3}{\sqrt{21.5^2 + 3.14^2 \times 8.33^2}} = 42.6 \text{ square feet.}$$

Example 54. **Fig. 30.**—The dimensions being the same as in the example 52.

The Vessel.

Depth of centre of gravity . $d = 2.8$ feet

Co-efficient . . . . . $\mathfrak{d} = 0.091$

Greatest immerse section . $\mathfrak{B} = 100$ square feet

Angle of resistance . . $\Phi = 6° 55'$

Require the values . . $\oplus = ?$

$$\oplus = sin.6° \, 55' \sqrt{100(1+0.091—sin.^2 6°55' \times cos.6°55')}$$
$= 1.2.$

$$\oplus = \sqrt{\frac{\sqrt{100} \; (0.1515 + 2)(100 + 4 \times 10^2)}{1160 \times 10}} = 0.96$$

$$\oplus = 1.2 + 0.96 = 2.16.$$

The Propeller.

Depth of centre of gravity .	$d = 5$ feet
Co-efficient	$\delta = 0.1515$
Acting area	$A = 42.6$ square feet
Require the value . . .	$\bigcirc = ?$

$$\bigcirc = \sqrt{42.6 \; (1 + 0.1515)} = 7$$

Require the slip $S = ?$

$$S = \frac{2.16}{7 + 2.16} = 0.23 \text{ or } 23 \text{ per cent.}$$

Example 55. **Fig. 31.**—Dimensions of the vessel being

Length in the load-line . .	$l = 100$ feet
Beam	$b = 20$ feet
Draft of water . . .	$\eth = 10$ feet
Depth of centre of gravity .	$d = 3.8$ feet
Greatest immerse section .	$\mathfrak{N} = 157$ square feet
Tonnage of displacement .	$Q = 258$ tons
Capacity	$C = 9000$ cubic feet
Require the angle of resistance	$\Phi = ?$

$$r = \frac{100^2 \times 10}{8 \times 3.8 \times 20} + \frac{3.8 \times 20}{2 \times 10} = 167.8 \text{ feet,}$$

$$w = \frac{200 \times 9000 \times 3.8}{157 \times 10 \times 167.8} = 26°,$$

See column . . $x\,\Phi$ and w } Table
Angle of resistance $\Phi = 13°\ 22'$ the answer **26°** } III.

Example 56. **Fig. 31.**—Dimensions of the propeller being

Diameter	$D = 8.33$ feet
Pitch	$P = 18.5$ feet
Require the acting area . .	$A = ?$ in square feet.

$$A = \frac{2.5 \times 8.33^3}{\sqrt{18.5^2 + 3.14^2 \times 8.33^2}} = 44.7 \text{ square feet.}$$

Example 57. **Fig. 31.**—Dimensions of the vessel being the same as in Example 55.

The Vessel.

Depth of centre of gravity .	$d = 3.8$ feet
Co-efficient	$\delta = 0.12$
Greatest immerse section .	$\mathfrak{B} = 157$ square feet
Angle of resistance . .	$\Phi = 13°\ 22'$
Require the values . .	$\oplus = ?$

$$\oplus = sin.13°\ 22'\ \sqrt{157\,(1 + 0.12 - sin.^2 13°\ 22' \times cos.13°\ 22')} = 2.39,$$

$$\oplus = \sqrt{\frac{\sqrt{100}\ (2 + 0.1515)\,(157 + 4 \times 10^2)}{1160 \times 10}} = 1.15,$$

$$\oplus = 2.39 + 1.15 = 3.54.$$

Propeller.

Depth of centre of gravity .	$d = 5$ feet
Co-efficient	$\delta = 0.1515$
Acting area	$A = 44.7$ square feet
Require the value . .	$\bigcirc = ?$

$$\bigcirc = \sqrt{44.7\ (1 + 0.1515)} = 7.2.$$

Require the slip $S = ?$

$$S = \frac{3.54}{7.2 + 3.54} = 0.33, \text{ or } 33 \text{ per cent.}$$

Example 58. **Fig. 32.**—Dimensions of the vessel being

Length in the load-line .	$l = 100$ feet
Beam in the load-line . .	$b = 20$ feet
Draft of water . . .	$\mathfrak{d} = 10$ feet
Depth of centre of gravity	$d = 4.66$ feet
Greatest immerse section .	$\mathfrak{X} = 183$ square feet
Tonnage of displacement .	$Q = 350$ tons
Capacity of displacement .	$C = 12200$ cubic feet
Require the angle of resistance	$\Phi = ?$

$$r = \frac{100^2 \times 10}{8 \times 4.66 \times 20} + \frac{4.66 \times 20}{2 \times 10} = 138.66 \text{ feet.}$$

$$w = \frac{200 \times 12200 \times 4.66}{183 \times 10 \times 138.66} = 44.75°.$$

See column . . .	$x\ \Phi$	and $w°$	Table III.
From . . .	22° 20′	" **45°**	
Subtract . . .	21° 53′	" **44°**	

Multiply . $0.75 \times 0° 27' = 12.75$, say $13'$,
To . . . . $21°53'$
Add . . . . $0° 13'$
Angle of resistance $\Phi = 22° \ 6'$.

Example 59. **Fig. 32.**—Dimensions of the vessel being

Diameter . . . . . $D = 8.33$ feet
Pitch . . . . . . $P = 16$ feet
Require the acting area . . $A = ?$ in square feet

$$A = \frac{2.5 \times 8.33^3}{\sqrt{16^2 + 3.14^2 \times 8.33^2}} = 50 \text{ square feet.}$$

Example 60. **Fig. 32.**—Dimensions of the vessel being the same as in Example 58.

The Vessel.

Depth of centre of gravity . $d = 4.66$ feet
Co-efficient . . . . . $\delta = 0.143$
Greatest immerse section . $\mathfrak{X} = 183$ square feet
Angle of resistance . . $\Phi = 22° \ 6'$
Require the values . . $\oplus = ?$

$$\oplus = sin.22° \ 6' \sqrt{183(1+0.143-sin.^2 22°6' \times cos.22°6')} = 7.$$

$$\oplus = \sqrt{\frac{\sqrt{100}\ (2 + 0.1515)(183 + 4 \times 10^2)}{1160 \times 10}} = 1.04,$$

$$\oplus = 7 + 1.04 = 8.04.$$

The Propeller.

Depth of centre of gravity . $d = 5$ feet
Co-efficient . . . . . $\delta = 0.1515$ feet
Acting area . . . . . $A = 50$ square feet
Require the value . . . $\bigcirc = ?$

$$\bigcirc = \sqrt{50\ (1 + 0.1515)} = 7.6.$$

Require the slip $S = ?$

$$S = \frac{8}{7.6 + 8} = 0.51 \text{ or } 51 \text{ per cent.}$$

When the dimensions of the vessel are given, the number of horse-power H which is required to drive the vessel M miles per hour, will be when

$$R = \mathfrak{A}\ v^2 \, sin.^2\Phi\ (1 + \delta - sin.^2\Phi\ cos.\Phi),$$

and

$$M = \frac{60 \times 60\ v}{5280} = \frac{v}{1.465},$$

of which

$$v = 1.465\ M,$$

and

$$v^2 = 2.15\ M^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

and

$$H = \frac{R\ 60\ v}{33000} = \frac{R\ v}{550},$$

of which

$$R = \frac{550\ H}{v} = \frac{550\ H}{1.465\ M} = \frac{375\ H}{M}, \quad . \quad . \quad (f)$$

Those values of v^2 and R, from (e) and (f), inserted in the formula (56) will be

$$R = \frac{375\,H}{M} = 2.15\,M^2\,\aleph\,sin.^2\Phi\,(1 + \delta - sin.^2\Phi\,cos.\Phi),$$

and

$$H = \frac{2.15\,M^3}{375}\,\aleph\,sin.^2\Phi\,(1 + \delta - sin.^2\Phi cos.\,\Phi),$$

lately

$$H = \frac{M^3}{174.3}\,\aleph\,sin.^2\Phi\,(1 + \delta - sin.^2\Phi\,cos.\Phi), \qquad (64)$$

This will give the horse-power *less* than what actually occurs in practice, owing to the *square* of the *sine*. A broken exponent would be more correct, but the formulæ in connection with the *only* Table III. are incomplete; therefore, we will rather retain the exponent *square* of the *sine*. When the formulæ are accompanied with the tables before spoken of, the trigonometrical signs and exponents will disappear; then the calculations will be simple and the results *correct;* although it will be seen, in the following examples, that the present calculation gives the result very near the fact. To the formula (64) must be added the friction in horse-power, which will be obtained by multiplying the co-efficient 1160 formula (62) by 174.3, and insert M^3 instead of v^2; as

$$H = \frac{M^3\,\sqrt{l}\,(\delta + 2)\,(\aleph + 4\,\mathfrak{d}^2)}{202500\,\mathfrak{d}}, \quad . \quad . \quad (65)$$

Example 61. **Fig. 30.**—The vessel to have the same dimensions as in the Example 52.

Greatest immerse section . $\mathfrak{N} = 100$ square feet
Angle of resistance . . $\Phi = 6° \ 55'$
Co-efficient . . . $\delta = 0.091$
Speed of the vessel to be . $M = 15$ miles per hour
Require the number of horse-power $H = ?$

$$H = \frac{15^3 \times 100 \times sin.6° \ 55'}{174.3} (1 + 0.091 - sin.^2 6° \ 55' \times$$

$cos.6° \ 55') = 30.25$ horses.

Friction

$$H = \frac{15^3 \times \sqrt{100} \ (+ \ 2)0.1515 \ (100 + 4 \times 10^2)}{202500 \times 10} =$$

17.9 horses.

Required horses $H = 30.25 + 17.9 = 53.15$, the answer.

Example 62. **Fig. 31.**—The vessel to have the same dimensions as in the Example 55.

Greatest immerse section . $\mathfrak{N} = 157$ square feet
Angle of resistance . . $\Phi = 13° \ 22'$
Co-efficient . . . $\delta = 0.12$
Speed of the vessel to be . $M = 12$ miles per hour
Require the number of horse-power $H = ?$

$$H = \frac{12^3 \times 157 \times sin.^2 13° \ 22'}{174.3} (1 + 0.12 - sin.^2 13° \ 22' \times$$

$cos.13° \ 22') = 87$ horses.

Friction

$$H = \frac{12^3 \sqrt{100} (2 + 0.1515)(157 + 4 \times 10^2)}{202500 \times 10} = 10.2 \text{ horses.}$$

Required horses $H = 87 + 10.2 = 97.2$, the answer.

Example 63. **Fig. 32.**—The vessel to have the same dimensions as in the Example 58.

Greatest immerse section . $\mathfrak{A} = 183$ square feet
Angle of resistance . . $\Phi = 22° \ 6'$
Co-efficient . . . $\delta = 0.143$
Speed of the vessel to be . $M = 10$ miles per hour
Require the number of horse-power $H = ?$

$$H = \frac{10^3 \times 183 \times sin.^2 22° \ 6'}{174.3} \left(1 + 0.143 - sin.^2 22° \ 6' \times cos. 22° \ 6'\right) = 150 \text{ horses.}$$

Friction

$$H = \frac{10^3 \sqrt{100} \,(2 + 0.1515)(183 + 4 \times 10^2)}{202500 \times 10} = 6.2.$$

Required horses $H = 150 + 6.2 = 156.2$, the answer.

Number of revolutions of the propeller will be per minute

$$n = \frac{88 \, M}{P\,(1 - S)}.$$

Fig. 30. $$n = \frac{88 \times 15}{21.5\,(1 - 0.196)} = 76.$$

Fig. 31. $$n = \frac{88 \times 12}{18.5\,(1 - 0.29)} = 93.$$

Fig. 32. $$n = \frac{88 \times 10}{16\,(1 - 0.50} = 110.$$

Tonnage.

The United States Custom House measurement for tonnage of vessels, is expressed by the formulæ.

$$T = \frac{b \, \mathfrak{d}}{95} \left(l - \tfrac{3}{5}\, b\right)$$

in which

$T =$ Tonnage of the vessel.

$b =$ Extreme beam in feet, taken above the mean vales.

$\mathfrak{d} =$ Depth of the vessel in feet. In double-decked vessels, half the beam b is taken as the depth. For single-decked vessels, the depth is taken from the under side of the deck-plank, to the ceiling of the hold.

$l =$ Length of the vessel in feet, from the fore-part of the stem to the after-part of the stern-post, measured on the upper deck.

By this rule, the tonnage of the vessels represented and exemplified by the **Figs. 30, 31,** and **32**, will be equal, and

$$T = \frac{20 \times 10}{95}\left(106 - \tfrac{3}{5}\, 10\right) = 210 \text{ Tons};$$

this appears to be an incomplete rule.

Machinists require the tonnage more accurately, when making contracts for capability of engines and propellers, in reference to the speed of the vessels.

The tonnage of the displacement will be found very near by,

$$Q = \mathfrak{B}\, l \times \begin{cases} 0.015 \text{ very sharp vessels,} \\ 0.020 \text{ very full vessels,} \end{cases}$$

in which

$Q =$ Tonnage of the displacement.

$\mathfrak{B} =$ Greatest immerse section in square feet.

$l =$ Length in the load line in feet.

For ordinary vessels, take the co-efficient $= 0.017$.

The results from the last examples for the figures on Plate XXXI. are collected in this

TABLE IV.

VESSEL.		Fig. 30.	Fig. 31.	Fig. 32.	
Length at the load line	l	100	100	100	feet.
Beam at the load line	b	20	20	20	"
Draft of water	ð	10	10	10	"
Depth of centre of gravity	d	2.8	3.8	4.66	"
Greatest immerse section	ℵ	100	157	183	square feet.
Custom-house measurem.	T	210	210	210	tons.
Tonnage of displacement	Q	143	258	350	"
Capacity of displacement	C	5000	9000	12200	cubic feet.
Radii of the circle arc	r	226	167.8	138.66	feet.
Angle of the circle arc	w°	12° 4′	26° 45′	44° 75′	degrees.
Angle of resistance	Φ	6° 55′	13° 22′	22° 6′	"
Speed of the vessels	M	15	12	10	miles per h.
Power required	H	53.15	97.2	158.2	horses.
PROPELLER.					
Diameter	D	8.33	8.33	8.33	feet.
Pitch	P	21.5	18.5	16	"
Acting area	A	42.6	44.7	50	square feet.
Depth of centre of gravity	d	5	5	5	feet.
The values	$\oplus$	7.2	2.39	7	velocities.
The values	$\bigcirc$	7	7.2	7.6	"
Slip	S	23	33	51	per cent.

The Steamer S. S. Lewis.

Example 53.—The dimensions on the S. S. Lewis will be found on page 95, viz.:—

Length in the load line . $l = 200$ feet

Beam . . . . . . $b = 30$ feet

Loaded to a draft of water . ð $= 15$ feet

Depth of centre of gravity . $d = 6.8$ feet

Plate XXXI.

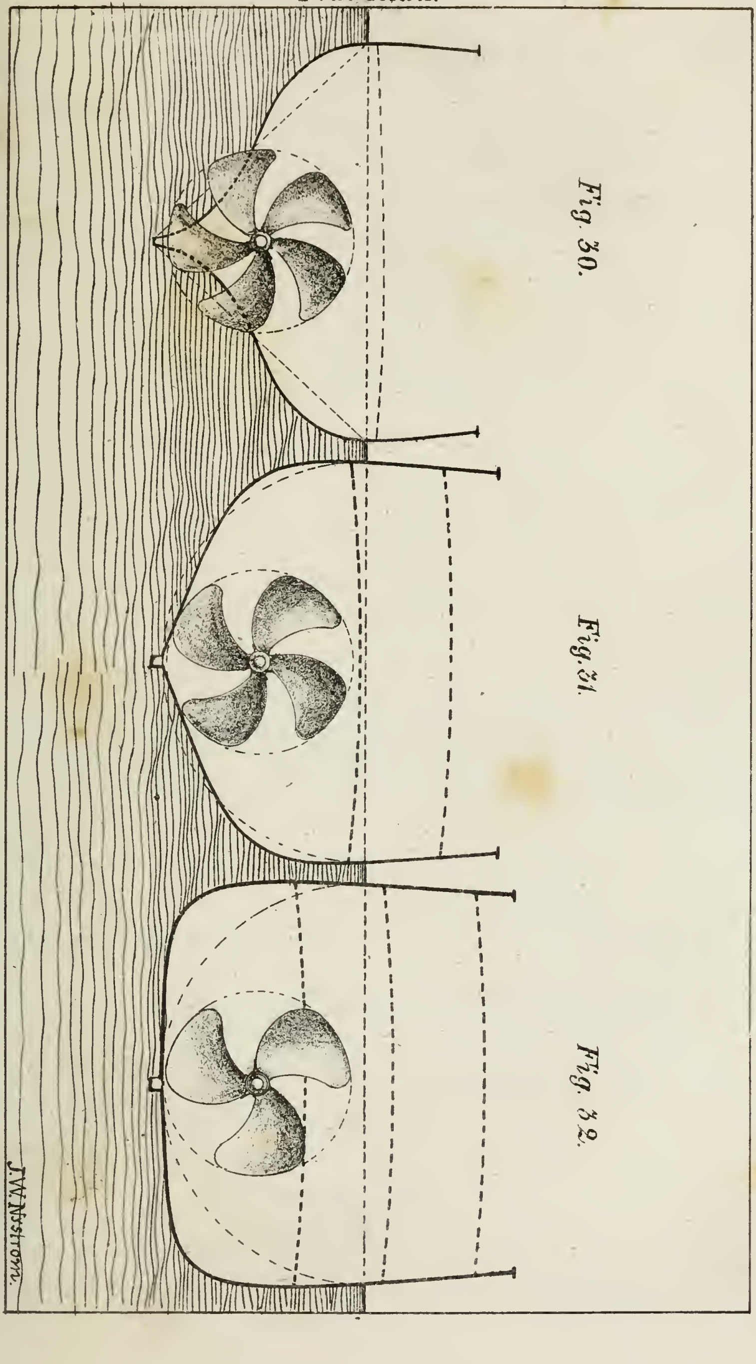


Fig. 30.

Fig. 31.

Fig. 32.

Greatest immerse section . $\mathfrak{N} = 400$ square feet
Tonnage of displacement . $Q = 1293$ tons
Capacity of displacement . $C = 45100$ cubic feet
Require the angle of resistance $\Phi = ?$

$$r = \frac{200^2 \times 15}{8 \times 6.8 \times 30} + \frac{6.8 \times 30}{2 \times 15} = 374.3 \text{ feet},$$

$$w^\circ = \frac{200 \times 45100 \times 6.8}{400 \times 15 \times 374.3} = 27.3^\circ.$$

See column . . .	$x\ \Phi$	and w	Table III.
From	$14^\circ\ 20'$	" **28°**	
Subtract . . .	$13^\circ\ 57'$	" **27°**	

Multiply . . $0.3 \times 0^\circ\ 23' = 7$ nearly
To . . . . $13^\circ\ 57'$
Add . . . . $0^\circ\ \ 7'$

Angle of resistance $\Phi = 14^\circ\ \ 4'$ the answer.

Example 54.—Dimensions of the S. S. Lewis propeller are

Diameter . . . . . $D = 13$ feet
Pitch . . . . . . $P = 33$ feet
Require the acting area . $A = ?$ in square feet.

$$A = \frac{2.5 \times 13^3}{\sqrt{33^2 + 3.14^2 \times 13^2}} = 104.5 \text{ square feet.}$$

Example 55.—The steamer S. S. Lewis and her propeller brought into action.

The Vessel.

Depth of centre of gravity . $d = 6.8$ feet
Co-efficient . . . . $\delta = 0.196$ feet

Greatest immerse section . $\mathfrak{X} = 400$ square feet
Angle of resistance . . $\Phi = 14° 4'$
Require the values . . $\oplus = ?$

$$\oplus = sin.14° 4' \sqrt{400\,(1 + 0.196 - sin.^2 14° 4' \times cos.14° 4')}$$
$$= 5.2,$$

$$\oplus = \sqrt{\frac{\sqrt{200}\,(2 + 0.196)(400 + 4 \times 15^2)}{1160 \times 15}} = 1.5,$$

$$\oplus = 5.2 + 1.5 = 6.7.$$

The Propeller.

Depth of centre of gravity . $d = 7.5$ feet
Co-efficient . . . . $\delta = 0.211$
Acting area . . . . $A = 104.5$ square feet
Require the value . . $\bigcirc = ?$

$$\bigcirc = \sqrt{104.5\,(1 + 0.211)} = 11.2.$$

Require the slip $S = ?$

$$S = \frac{6.7}{11.2 + 6.7} = 0.37, \text{ or } 37 \text{ per cent.}$$

DESCRIPTION

OF A

CALCULATING MACHINE.

PATENTED BY J. W. NYSTROM.

PHILADELPHIA, MARCH 4, 1851.

PLATE XXXII. This is a calculating machine, by which all the calculations in this work are computed. It is a round plate, on which are fixed two movable arms; by moving those arms, the calculations are computed. The machine has been *in use* four years by the author of this book, who is the inventor, and has thoroughly tested its practical utility.

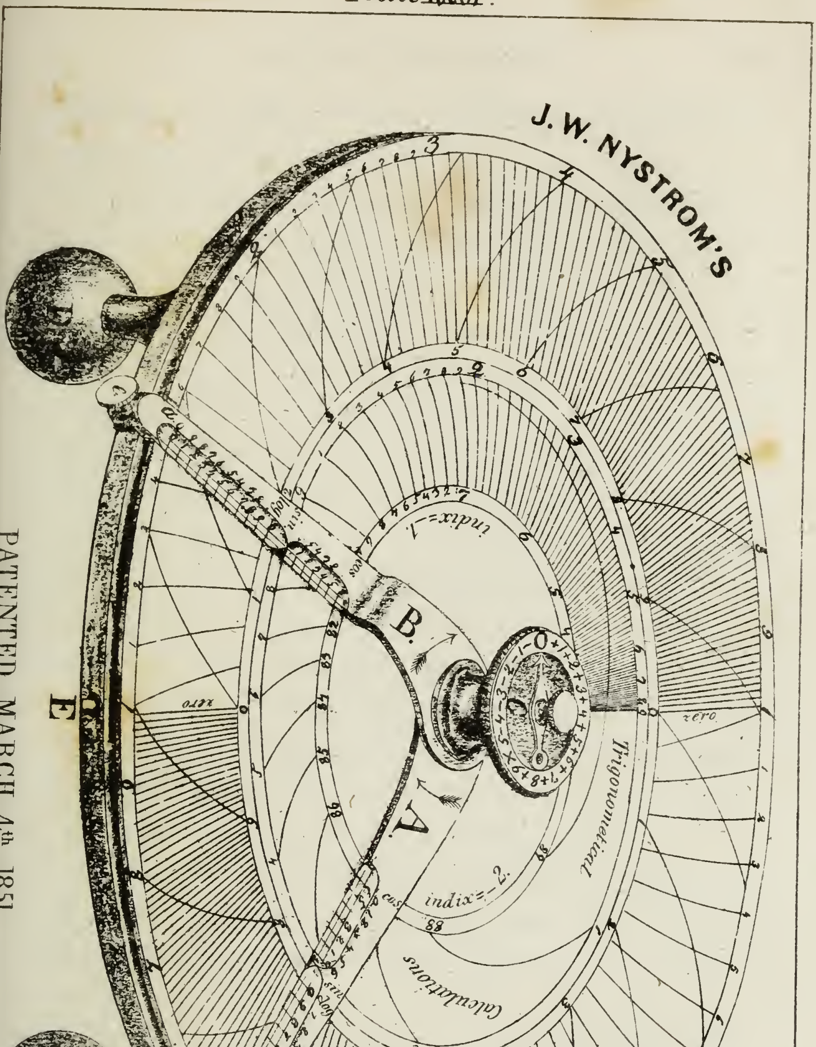
On the round plate are engraved a number of curved lines, in such a form and divisions, that, in the construction or manufacturing of it, a difficulty represented itself, viz.: the need of a correct instrument to draw the curved lines, which are laid out in progressive divisions. Such an instrument has been completed, and one which can be relied on for its accuracy. The first division was done by the dividing-machine, in the office of the Coast Survey, at Washington, which is perhaps the best in the world. The facility with which the calculating machine now can be manufactured, enables the patentee to furnish them at greatly reduced prices. The calculating machine was exhibited at the Franklin Institute Exhibition in the year 1849. Since then, it has been patented, and several improvements made upon it.

DESCRIPTION.

The Calculating Machine consists of a round disk of metal or any other suitable material. It has two graduated arms *A* and *B*, extending from the centre *C*, to the periphery. On the outer end of each arm is a screw *e*, for the purpose of fastening the arms in any particular place on the disk. In the centre is a screw *C*, to clamp the two arms *A* and *B* together; when clamped, they can be moved freely around the centre *C*.

The operation of calculation is performed by moving these two arms together and independently; for each operation, the arm *B* must be set in one particular place on the disk called *zero*. In order to be correct and save time, and to facilitate the setting of the arm *B* on *zero*, there is placed a nail *E*, which is to be operated when the arm *B* approaches *zero*; this is done by keeping one finger on a spring attached to the nail *E*, under the dish; pinch the spring with the finger, the nail *E* will project over the periphery, and

Plate XXXII.


J. W. NYSTROM'S
CALCULATING MACHINE.
PATENTED MARCH 4th 1851.
Trigonometrical
calculations
indix=-1.
indix=-2.
zero
zero
B.
A.
E
D

the arm B stops at *zero.* A little practice, this is done with the greatest facility.

On the disk are engraved 290 different *curved lines ;* for each curved line, are calculated 10 points; by joining these, each curved line is obtained; that is, before the 290 curved lines can be drawn, 2900 points must be calculated. This is a laborious undertaking, but patient industry accomplishes it, and by it a machine is obtained, on which any question of calculation can be computed, no matter how complicated it may be.

The curved lines are laid out in two concentric rings on the disk; in the edges of the two rings are figure circles, which are marked *a, log., sin., cos.,* on the arm. The lines which are numbered in the circle *a* are for *multiplication* and *division, powers* and *extracting of roots.* The lines which are numbered in the circle *log.,* are for *addition* and *subtraction,* or the logarithm for the numbers in the circle *a.*

The circle *a* is divided into a logarithmic series, so that the distances between the lines and numbers, being in proportion as the logarithm for the number counted from *zero.*

Circle *log.* is divided in an arithmetical series, where the difference between the terms is = 1, then by starting the two divisions of the circles *a* and *log.,* from a common point or radius, the circle *log.* will be the logarithm for the circle *a.* It is not necessary that this commencing line should be in the direction of the radius, but on this instrument it is done so.

Before going further, it may be well to define what is meant by logarithm.

LOGARITHM *is an exponent of a power, to which* 10 *must be raised to give a certain number.* This exponent is called the logarithm for the number, which will be understood by,

	Number.		Logarithm.		Base.	Exponent.		
logarithm for	100	$=$	2	because	10	2	$=$	100.
log.	1000	$=$	3	"	10	3	$=$	1000.
log.	10000	$=$	4	"	10	4	$=$	10000.
log.	616600	$=$	5.79	"	10	5.79	$=$	616600.

The unit of the logarithm is called the *characteristic* or *index,* and the decimal part is called the *mantissa;* the sum of the *index* and mantissa is the logarithm, as,

$$\begin{aligned} &\textit{index}\ 616600 = 5 \\ &\textit{mant.}\ 616600 = 0.79 \\ &\textit{log.}\ 616600 = 5.79 \end{aligned}$$

The invariable number 10 is called the base for the system of logarithm. It is not necessary that the base should be 10; it can be any number; but as our system of arithmetic has 10 to the base, it is more convenient to have the same base for the system of logarithms. All the tables of logarithm now in common use are calculated with 10 to the base. But whatever the base may be, the nature of the logarithms remains the same; which is, the logarithm for the base = 1, and logarithm for 1 = 0.

Logarithms are principally used in *Multiplication*, *Division*, *Powers*, and *Extracting of Roots*, which is executed by the following four *simple rules*.

MULTIPLICATION.

RULE 1. *Add together the logarithms for the factors; the sum is the logarithm for the product.*

Example.

$$3456 \times 50. \quad \begin{array}{ll} \textit{log.}\ 3456 & = 3.53857 \\ \textit{log.}\ \ \ \ 50 & = \underline{1.69897} \\ \textit{sum} & = 5.23754 = \textit{log.}\ 172800. \end{array}$$

the product.

DIVISION.

RULE 2. *Subtract the logarithm for the divisor, from the logarithm for the dividend; the difference is the logarithm for the quotient.*

Example.

$$172800 : 50. \quad \begin{array}{ll} \textit{log.}\ 172800 & = 5.23754 \\ \textit{log.}\ \ \ \ \ \ \ \ 50 & = 1.69897 \\ \textit{difference} & = 3.53857 = \textit{log.}\ 3456. \end{array}$$

the quotient.

POWERS.

Rule 3. *Multiply the logarithm for the number, by the exponent of the power; the product is the logarithm for the power of the number.*

Example.—150^3 $log.\ 1.50^3 = 2.176091 \times 3 =$
$6.528273 = log.\ 3375000 = 150^3$.

EXTRACTING OF ROOTS.

Rule 4. *Divide the logarithm for the number, by the index, for the root; the quotient is the logarithm for the root of the number.*

Example.—$\sqrt[3]{3375000}$. $log.\ 3375000 = 6.528273 :$
$3 = 2.176091 = log.\ 150 = \sqrt[3]{3375000}$.

This is the principle upon which the *calculating machine is based,* and the addition and subtraction of the logarithm for the numbers, are computed without noticing the logarithm; only by moving the arms *A* and *B,* together and independently, by the intersections of the curved lines the *result* is obtained. It is not necessary to understand the nature of logarithms to use the instrument, it only requires attention to the *simple rules* which here will be given.

Circle a.—In the circle *a* are two sizes of figures, of which the larger one represents the *first* figure of a number, and the small one the *second;* for instance

28 is a number composed of two figures; 2 is the *first figure*, and 8 the *second*.

Example.—To set the arm *A* on the number 28 (circle *a*). Set the arm *A* on the large 2, move the arm further, until it comes to the small 8 (the eighth line from the large 2), fasten the arm *A* with the screw *e*, then the arm *A* is set on 28; but this 28 can represent any multiple or parts by 10, for instance 0.0028, 0.28, 2.8, 28, 280, 28000, &c.

When the number contains more than two figures, as 2835, the third and fourth figures are to be found on the arm, where it intersects the curved line; the third figure is the small figure on the arm counted from *a*, and the fourth figure is represented by the small divisions between the figures on the arms. When the arm stands on 28, move it further, until it intersects the eighth curved line at 3 on the arm, then the arm stands on 283; move it a little further until the same curved line intersects the fifth division between 3 and 4 on the arm, then the arm stands on 2835. In the same manner the arm is to be set on any other number, which, by a little practice, is done *instantly*.

Circle log.—This circle contains the logarithm for the numbers in the circle *a*. When the arm is set on a number circle *a*, at the same time the arm intersects a curved line belonging to the *circle log.*, this latter intersection is the decimal part (mantissa) of the logarithm for the number in the circle *a*.

Example. *Log.* 7 = 0.845.—Set the arm *A* on 7, *circle a,* the mantissa in the *circle log.* is 845; the first figure 8, is the 8 curved line, numbered in *circle log.* which intersects the arm, the second figure 4 is the fourth figure on the arm, counted from *log.*, and the third figure 5, is the fifth division between 4 and 5 on the arm.

The nature of logarithms in connection with their numbers is such, that the index for the logarithm is always *one* less than the *number* of figures in the number for which the logarithm is to be found (when the base of the logarithm is 10).

Index 60 = 1	because	60 is two figures	and	2 — 1 = 1
616 = 2	"	616 " three	"	3 — 1 = 2
6166 = 3	"	6166 " four	"	4 — 1 = 3
61663000 = 7	"	61663000 " seven	"	8 — 1 = 7

In difficult calculations, combined with *powers* and *roots,* a correct account must be kept on these indices, in order to be certain of the number of figures in the result. For that purpose there is a small *hand* on the screw *C,* which is to be operated separately for each operation by the arms. For multiplication, *add the indices by the small hand.* For division, *subtract the index for the divisor, from the index for the dividend,* then when the operations are finished, the small *hand* shows the index for the result; adding one to it gives the number of figures in the result. If the index becomes negative, the result is a corresponding decimal fraction, and the *hand* shows how many 0 it is before the figures, including the index 0.

Example.—If the arm shows 28, on circle *a*, and the *hand* shows the index + or —, as

Index	— 3	the decimal fraction is		0.0028
"	— 2	"	"	0.028
"	— 1	"	"	0.28
"	= 0	the corresponding number is		2.8
"	+ 1	"	"	28.
"	+ 2	"	"	280.
"	+ 4	"	"	28000.
"	+ 7	"	"	28000000.
	&c.			&c.

Abbreviations.

Set *A* on, means set the arm *A* on.
Set *B* on, " set the arm *B* on.
Fasten *Ae*, " fasten the arm *A* with the screw *e*.
Fasten *Be*, " fasten the arm *B* with the screw *e*.
Clamp *C*, " clamp the two arms with the screw *C*.

MULTIPLICATION.

Multiplication is to be computed on the circle *a*, and without any exception follows this simple rule 1. Multiply two numbers together.

RULE 1. *Set A on one of the numbers, fasten Ae. Set B on zero, clamp C, loose Ae. Move the arms until B comes to the other number, then the arm A shows the product.*

Example 1.—Multiply the number 3 by 2. Set A on 3, fasten Ae; set B on *zero*; clamp C; loose Ae; move the arms until B comes to 2; then A shows the product 6.

Example 2.—Multiply 436000 by 12500. Set A on 436000, fasten Ae; set B on *zero*; clamp C; loose Ae; move the arms until B comes to 12500; then A shows the product 5450000000.

When the arm A is set on 436, set the *hand* on its index 5; when the arms are moved until B comes to 125, add its index 4 to 5, which will be 9, the index for the result. These operations are accomplished instantly.

When there are more than two factors to be multiplied together, consider the product of two factors as one factor, and continue the multiplication with the next factor, as before described. It matters not how many factors there may be, the multiplication can be continued to any extent, and *all the products that come between the operation need not be observed, only the last product or the result.* This very point constitutes a superiority of the instrument *over* all the calculating machines ever invented.

Example 3.—Multiply $150 \times 360 \times 4550 = 245700000$. Set A on 150; B on *zero*; clamp C; move the arms until B comes to 360; fasten Ae; loose C; set B on *zero*; clamp C; loose Ae; move the arms until B comes to 4550; then A shows the product 245700000.

It will be seen in this example, that the product of

150, and 360, was not noticed; but the multiplication was continued by 4550, and *first* noticed the product of the *three* factors, and so on how many factors there may be.

DIVISION.

Division is to be computed on the same *circle a*, and is only a reverse of the operation for multiplication, which will be seen in its *general rule* 2. To divide one number in another.

RULE 2. *Set A on the dividend; B on the divisor; clamp C; move the arms until B comes to zero; then A shows the quotient.*

Example 1.—Divide 378 by 9. Set *A* on 378; *B* on 9; clamp *C*; move the arms until *B* comes to *zero;* then *A* shows the quotient 42.

Example 2.—Divide 503800 by 680. Set *A* on 503800; *B* on 680; clamp *C*; move the arms until *B* comes to *zero;* then *A* shows the product = 740.

When the dividend contains more than one factor, multiply the factors together by *Rule* 1, and divide by the divisor according to *Rule* 2.

Example 3.—If 343 men can do a certain work in 64 days, how many days will it take for 196 men to do the same work?

$$\frac{343 \times 64}{196} = 112 \text{ days.}$$

Set *A* on 343; *B* on 196; clamp *C*; move the arms until B comes to 64: then A shows the answer = 112 days; that is a computation of three qualities accomplished in one single operation.

When the divisor contains more than one factor, divide by the first factor as aforesaid, consider the quotient as a new dividend, and continue the division by the next factor.

Example 4.—How many days will it take for a steamboat from New York to Liverpool, running 12 miles per hour, the distance being 3100 miles?

$$\frac{3100}{12 \times 24} = 10{,}76 \text{ days.}$$

Set A on 3100; B on 19; clamp C; move the arms until B comes to *zero;* fasten Ae; loose C; set B on 24; clamp C; loose Ae; move the arms until B comes to *zero*; then A shows the answer =10.76 days.

When both the dividend and divisor contain a number of factors, the factors will be separated by a character $\int$ which is suitable for the operation by the machine.

Example 5.—A ship's crew of 300 men were so supplied with provisions for 12 months, that each man was allowed 30 ounces per day; but, after being out 6 months, they found it would require 9 months more to finish it, and 50 of their number were lost. It is required to find the daily allowance of each man during the last nine months.

$$\frac{300 \times 30\ (12 - 6)}{9\ (300 - 50)} = \frac{300}{9} \int \frac{30}{250} \int \frac{6}{-} = 24 \text{ ounces.}$$

Set A on 300; B on 9; clamp C. Now, this character means, that the arm B, which stands on 9, is to be moved to 30; again set the arm B on 250, and move it to 6, then A shows the answer = 24 ounces.

Example 6.—If 288 men, in 5 days, working 11 hours a day, can dig a trench 220 yards long, 3 yards wide and 9 yards deep; in how many days, working 9 hours a day, will 24 men dig a trench 240 yards long, 7 yards wide, and 3 yards deep?

$$\frac{288}{220} \int \frac{5}{3} \int \frac{11}{2} \int \frac{240}{24} \int \frac{7}{9} \int \frac{3}{-} = 280 \text{ days.}$$

This is to be computed in the same manner as in the example 5; viz.: Set A on 280; B on 220; clamp C; move B to 5; set B on 3, and move it to 11, and so on until the arm B arrives at the last factor 3; then A shows the answer = 280 days.

It is evident that the arm A must be fastened between each operation, but it need not be noticed wherever the arm A stops, until the result comes, and the whole operation with the *eleven quantities* is accomplished in less than half a minute. It does not take more time if the numbers are broken to fractions, as it is just as easy to set the arm on $7\frac{934}{1000}$ as it is to set it on 7.

PROPORTION.

The manœuvring of *proportions,* is a most astonishing feature by this instrument. Suppose we have the proportion as 2 : 3. Set *B* on 2; *A* on 3; clamp *C*. Now, wherever the arms will be placed on the disk, the numbers at *B* and *A* will always be in the proportion as 2 : 3. Set *B* on 6, the arm *A* will show 9. Set *A* on 12, the arm *B* will show 8. Set *B* on $13\frac{29}{100}$, the arm *A* will show $19\frac{93}{100}$, &c. &c. &c.

In whatever proportion the arms are once clamped, it will remain the same in any other position on the disk.

Interest.

Example 1.—What is the interest on 765 dollars, at 6 per cent. per annum. Set *A* on 6 per cent., *B* on *zero*, clamp *C*, move the arms until *B* comes to 765, then *A* shows the interest = \$45.90.

On whatever number of dollars the arm *B* is set, the arm *A* will show the interest for one year; or the arm *A* be set on any interest, *B* will show the sum of money.

The interest to be found for any number of years, months, weeks, or days.

Let the arm *A* remain on the interest for one year.

$$\text{Set } B \text{ on} \left\{ \begin{array}{llll} \textit{zero}, & \text{if number of} & & \text{years,} \\ 12 & \text{“} & \text{“} & \text{months,} \\ 52 & \text{“} & \text{“} & \text{weeks,} \\ 365 & \text{“} & \text{“} & \text{days,} \end{array} \right\} \text{Clamp } C.$$

Now for any number of years, months, weeks, or days, on the arm B, the corresponding interest will be found on the arm A.

It works in the same manner for *Rebate* and *Discount.*

To constructors, draughtsmen, this will be found a most valuable assistance. When changing a machine, or drawing from one scale to another, *any* scales are instantly to hand on this machine, by once placing the arms in the proportion of reduction; then any measure in feet and inches on one arm will show the reduced or increased measure on the other arm. To ship-builders, particularly, it will meet the highest approbation, when *enlarging* and *reducing* lines of vessel; and at the same time, the machine will cover all the ship-builder's calculation.

INVOLUTION.

The power of a number is only a multiplication by the number itself. The dignity of the power is marked by a small figure called *exponent* on the right of the number.

Example 1.—3^2 means that 3 is multiplied by itself, or $3^2 = 3 \times 3 = 9$; ^{2} is the exponent of 3, and $4^3 = 4 \times 4 \times 4 = 64$; ^{3} is the exponent.

To operate this by the machine is evidently the same as *rule* 1 for multiplication. When the exponent is large, as 3^8, we need *not* multiply by 3 eight times, because $3^8 = 9^4 = 81^2 = 6561$. Set A on 3, B on *zero,* clamp C, move the arms until B comes to 3, then A shows $9 = 3^2$. In the same manner multiply by 9, the arm A will show $81 = 9^2 = 3^4$, and 81 multiplied by 81, the arm A will show $6561 = 81^2 = 9^4 = 3^8$. If the exponent is an odd number, as

$$3^7 = \frac{3^8}{3} = \frac{6561}{3} = 2187$$

EVOLUTION.

Extraction of roots is the reverse of power; as the root of a number taken to the same power as the index for the root is coequal to the number. Roots are generally marked by the character $\sqrt{}$ in which the index of the root is placed as $\sqrt[3]{}$, for the square or second root, the index ^{2}, is always omitted.

Example 1.— $\sqrt{9} = 3$. This is to be computed by the circle a, in connection with the circle *log.* Set A on 9, the corresponding mantissa on the circle *log.* is 954; divide 954 by 3 = 318. Set B on mantissa 318, circle *log.*; it will correspond with 3 on circle a, which is the answer.

Example 2.—The lines of a vessel of 2000 tons are to be reduced to a smaller vessel of only 1400 tons. What will be the proportion of the scales for the lines?

$$\sqrt[3]{2000} : \sqrt[3]{1400} = 12.6 : 11.19.$$

Set A on 2000, mantissa on circle *log.*, is 301 divided by 3 = 1003. Set B on mantissa 1003, it will correspond with 12.6 on circle *a*. Set *A* on 1400, mantissa is 146 divided by 3 = 0487. Set *B* on mantissa 0487, it will correspond with 11.19 on circle *a*.

Set *A* on 126, clamp *C*. Now the arms are set on the two scales for the vessels, so that the arm *A* is the scale for the larger vessel, and *B* for the smaller. If the arm *A* be placed on any linear dimension of the larger vessel, the arm *B* will show the corresponding linear dimension of the smaller vessel.

Suppose the length of the larger vessel were 225 feet. Set *A* on 225, then *B* shows 200 feet, the length of the smaller. The beam of the largest vessel is 40 feet on *A*, is 35.5 on *B*, the beam of the smaller, &c.

Example 3.—$\sqrt[5]{7835000}$. Set *A* on 7835, the index is 6, mantissa on *circle log.* is 894, then 6.894 : 5 = 1.379. Set *A* on mantissa 379, the corresponding number on circle *a* is 23.93, the root.

To compute algebraical formulæ by the machine.—Set up the formula for the example, and insert the value of the corresponding quantities, as seen in the pages 72 and 73, and perform the operations as before described.

Example 1. *Formula* 17, page 80.—The length of a propeller is $L = 3$ feet, the angle $v = 42°$. Require the pitch $P = ?$

$$P = \frac{360\ L}{v} = \frac{360 \times 3}{42°} = 25.7 \text{ feet.}$$

Set *A* on 360, *B* on 42, clamp *C*. Move the arms until B comes to 3, then A shows the pitch $P = 25.7$ feet.

When there are a number of factors in both the numerator and denominator, it is most convenient to separate them by the characters described on page 190, and operate in the same manner. If there are any roots to extract, place the root marked in the first divisions as follows:—

Example 2. *Formula* 24, page 73.—The values of those quantities are, say

Number of revolutions . .	$n = 63$ per minute
Diameter of propeller . .	$D = 9$ feet 3 inches
Pitch of propeller . . .	$P = 24$ feet
Slip of propeller . . .	$S = 32$ per cent.
Diameter of cylinders . .	$d = 29\frac{1}{2}$ inches
Stroke of piston . . .	$s = 25$ inches
Friction	$f = 30$ per cent.
Require the effectual pressure	$p = ?$ in pounds.

$$p = \frac{\sqrt{P S}.50\, n\, D^2}{d^2\, s\, (1 - f)} = \frac{\sqrt{24 \times 0.32}}{(1 - 0.30)} \int \frac{50}{29.5} \int \frac{63}{29.5} \int \frac{9.25}{25} \int \frac{9.25}{1} = 49.2 \text{ pounds.}$$

Multiply 24 by 0.32, mantissa is 885 divided by $2 = 4425$. Set A on mantissa 4425. Subtract 0.30 from $1 = 0.70$. Set B on 0.70, clamp C, move the arms until B comes to 50, and proceed as described on page 190, until B arrives at the last factor 9.25, then A shows the effectual pressure $p = 49.2$ pounds per square inch.

Example 3. *Formula* 20, page 73.—The values of the quantities being the same as in the preceding example.

Require the diameter $d = ?$ in inches.

$$d = \sqrt[3]{\frac{\sqrt{PS}.50\ n\ D^2}{(1-f)\ p}} = \sqrt[3]{\frac{\sqrt{24 \times 0.32}}{(1-0.30)} \int \frac{50}{49.2} \int \frac{63}{1} \int \frac{9.25^2}{1}}$$

$= 27.8$ inches.

Set A on the mantissa 4425 (from the preceding example corresponding to $\sqrt{P\,S}$). Set B on 0.70, clamp C, move the arms until B comes to 50, and proceed as before described. Keep account of the indexes, and remember to make two operations by the factor 9.25. At the last operation the index will be 4 and mantissa 336. As the cube root is to be extracted, divide 4.336 by $3 = 1.445$.

Set A on the mantissa 445, the answer on circle a is $d = 27.8$ inches.

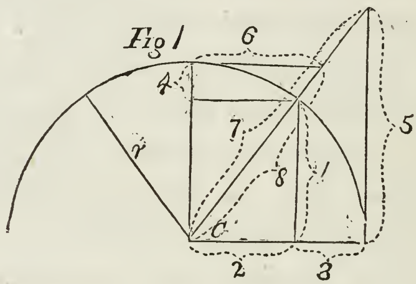
This example is computed in less than one minute.

TRIGONOMETRY.

Trigonometry is that part of geometry which treats of triangles. It is divided into two parts, viz., *plane* and *spherical*. Plane trigonometry treats of the triangles which are (or imagined to be) drawn on a plane. Spherical trigonometry treats on the triangles which are (or imagined to be) drawn on a sphere.

A triangle contains seven quantities, namely, three sides, three angles, and the surface. When any three of these quantities are given, the four remaining ones can by them be ascertained (one side or the area must be one of the given quantities), and the operation is called *solving the triangle*, which is only an application of arithmetic or geometrical objects.

For the foundation of the above-mentioned solution, there are assumed eight help quantities, which are called *trigonometrical functions*, which are the following in names and number, corresponding with the Fig. 1.



1.	*Sinus*,	abbreviated	*sin. C*
2.	*Cosinus*,	"	*cos. C*
3.	*Sinus-versus*,	"	*sinv. C*
4.	*Cosinus-versus*,	"	*cosv. C*
5.	*Tangent*,	"	*tan. C*
6.	*Cotangent*,	"	*cot. C*
7.	*Secant*,	"	*sec. C*
8.	*Cosecant*,	"	*cosec. C*

$r =$ *Radius* of the circle, which is the unit by which the functions are measured.

In the accompanying tables, these functions are calculated at every 10 minutes per degree in the quadrant of the circle represented by Fig. 1. The *circle arc* between the two lines 8 and 2, 3, is a measure of the angle for which the functions are mentioned. This angle is denoted by the letter C, and the expression $sin. C$ means the line 1 compared with the radii r at a given angle C.

Say the angle C to be 60°. In the first column of the table for the *sin.*, 60° corresponds with 0.86602 in the next column, which is the length of the *sin.* for 60° compared with the radii r as a unit, and the expression

$$sin. 60^\circ \times 36 \text{ means } 0.86602 \times 36 = 31.17672,$$

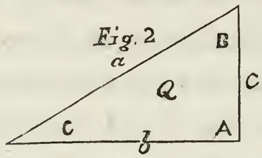
and likewise with all the other trigonometrical expressions.

In a triangle, these functions have a certain relation to the opposite side of the angle; it is this relationship which enables us to solve the triangle only by the application of simple arithmetic.

In a *triangle*, the sides are denoted by the letters a b c; their respective opposite angles are denoted by A, B, C, and the area by Q. The sides a, b, and c bear the following relation to the trigonometrical functions:—

Plane Trigonometry.

Fig. 2. *A right-angled triangle.*



$$a : c = 1 : sin. C, \text{ of which } a = \frac{c}{sin. C}, \quad . \quad (1)$$

$$a : b = 1 : cos. C, \quad \text{"} \quad b = a\, cos. C, \quad . \quad (2)$$

$$b : c = 1 : tan. C, \quad \text{"} \quad c = b\, tan. C, \quad . \quad (3)$$

$$sin. C : cos. C = tan. C : r.$$

Formula (1).—If the side c and the angle C are given, the side a will be found simply by dividing the side c by $sin. C$. Suppose the side c is 36 feet and the angle $C = 30°$. In the accompanying table for *sine*, 30° in the first column corresponds with 0.5000 in the next one, which is the length of the *sine* for 30°, and

$\frac{36}{sin. 30°} = \frac{36}{0.5000} = 72$ feet, the length of the side a.

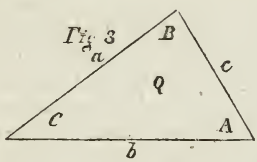
Formula (2).—The length of the side a and the angle C are given. Multiply the length a by $cos. C$, the product is the length of the side b. Suppose the side a is 325 feet long, and the angle $C = 42°$, in the table for *cosine*, marked on the bottom of the table, 42° in the last column corresponds with 0.74314 in the next column, which is the length of the *cosin.* for 42°, and

$325 \times cos.42° = 325 \times 0.74314 = 241.52$ feet, the length of the side b.

Formula (3). **Fig. 2.**—Let the line c represent the height of a steeple. From the centre at A is measured the horizontal line, $b = 285$ feet, to a point C, in which the angle to the top of the steeple is measured to be 38°. What will be the height of the steeple? See table for the tangent. In the first column, 38° corresponds with 0.78128 in the next column, which is the length of the *tangent* for 38°, and

$285 \times tan.38° = 285 \times 0.78128 = 222.5$ feet, the height of the steeple.

An oblique-angled triangle.



$$a : b = sin.A : sin.B, \text{ of which } a = \frac{b\ sin.A}{sin.B}, \quad (4)$$

$$b : c = sin.B : sin.C, \text{ of which } b = \frac{b\ sin.B}{sin.C}, \quad (5)$$

$$a : c = sin.A : sin.C, \text{ of which } c = \frac{a\ sin.C}{sin.A}, \quad (6)$$

$$sin.C = \frac{c\ sin.B}{b}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Examples for those formulæ will be computed by the machine, and without the tables. Without the machine, tables of this construction will be competent and useful for all mechanical calculations.

NATURAL SINE.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09294	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13340	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21643	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48480	61
29	.48480	.48735	.48988	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50502	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53729	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Degrees.

NATURAL COSINE.

NATURAL SINE.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71933	44
46	.71933	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75470	41
49	.75470	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77162	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86891	.87035	.87178	.87520	.87461	29
61	.87461	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91811	.91936	.92050	23
67	.92050	.92163	.92276	.92387	.92498	.92609	.92718	22
68	.92718	.92826	.92934	.93041	.93147	.93253	.93358	21
69	.93358	.93461	.93564	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95798	.95881	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97236	.97304	.97371	.97437	13
77	.97437	.97402	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	.99813	.99830	.99847	.99862	3
87	.99862	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.99996	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Degrees.

NATURAL COSINE.

TANGENT.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11098	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24207	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28045	.28359	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31849	.32170	.32491	72
18	.32491	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47340	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52426	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59296	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64527	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	.80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83909	50
40	.83909	.84406	.84906	.85408	.85912	.86414	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92704	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97532	.97699	.98269	.98843	.99419	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Degrees.

COTANGENT.

TANGENT.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2130	1.2203	1.2275	1.2348	39
51	1.2348	1.2422	1.2496	1.2571	1.2647	1.2722	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3679	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4732	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6752	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8939	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1608	2.1774	2.1942	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8769	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1652	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6011	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.760	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.074	19.081	3
87	19.081	20.205	21.470	22.003	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.289	1
89	57.289	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Degrees.

COTANGENT.

SECANT.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0001	89
1	1.0001	1.0002	1.0002	1.0003	1.0004	1.0005	1.0006	88
2	1.0006	1.0007	1.0008	1.0009	1.0010	1.0012	1.0013	87
3	1.0013	1.0015	1.0016	1.0018	1.0020	1.0022	1.0024	86
4	1.0024	1.0026	1.0028	1.0031	1.0033	1.0035	1.0038	85
5	1.0038	1.0040	1.0043	1.0046	1.0049	1.0052	1.0055	84
6	1.0055	1.0058	1.0061	1.0064	1.0068	1.0071	1.0075	83
7	1.0075	1.0078	1.0082	1.0086	1.0090	1.0094	1.0098	82
8	1.0098	1.0102	1.0106	1.0111	1.0115	1.0120	1.0124	81
9	1.0124	1.0129	1.0134	1.0139	1.0144	1.0149	1.0154	80
10	1.0154	1.0159	1.0164	1.0170	1.0175	1.0181	1.0187	79
11	1.0187	1.0192	1.0198	1.0204	1.0210	1.0217	1.0223	78
12	1.0223	1.0229	1.0236	1.0242	1.0249	1.0256	1.0263	77
13	1.0263	1.0269	1.0277	1.0284	1.0291	1.0298	1.0306	76
14	1.0306	1.0313	1.0321	1.0329	1.0336	1.0344	1.0352	75
15	1.0352	1.0360	1.0369	1.0377	1.0385	1.0394	1.0403	74
16	1.0403	1.0411	1.0420	1.0429	1.0438	1.0447	1.0456	73
17	1.0456	1.0466	1.0475	1.0485	1.0494	1.0504	1.0514	72
18	1.0514	1.0524	1.0534	1.0544	1.0555	1.0565	1.0576	71
19	1.0576	1.0586	1.0597	1.0608	1.0619	1.0630	1.0641	70
20	1.0641	1.0653	1.0664	1.0676	1.0687	1.0699	1.0711	69
21	1.0711	1.0723	1.0735	1.0747	1.0760	1.0772	1.0785	68
22	1.0785	1.0798	1.0810	1.0823	1.0837	1.0850	1.0863	67
23	1.0863	1.0877	1.0890	1.0904	1.0918	1.0932	1.0946	66
24	1.0946	1.0960	1.0974	1.0989	1.1004	1.1018	1.1033	65
25	1.1033	1.1048	1.1063	1.1079	1.1094	1.1110	1.1126	64
26	1.1126	1.1141	1.1157	1.1174	1.1190	1.1206	1.1223	63
27	1.1223	1.1239	1.1256	1.1273	1.1290	1.1308	1.1325	62
28	1.1325	1.1343	1.1361	1.1378	1.1396	1.1415	1.1433	61
29	1.1433	1.1452	1.1470	1.1489	1.1508	1.1527	1.1547	60
30	1.1547	1.1566	1.1586	1.1605	1.1625	1.1646	1.1666	59
31	1.1666	1.1686	1.1707	1.1728	1.1749	1.1770	1.1791	58
32	1.1791	1.1813	1.1835	1.1856	1.1878	1.1901	1.1923	57
33	1.1923	1.1946	1.1969	1.1992	1.2015	1.2038	1.2062	56
34	1.2062	1.2085	1.2109	1.2134	1.2158	1.2182	1.2207	55
35	1.2207	1.2232	1.2257	1.2283	1.2308	1.2334	1.2360	54
36	1.2360	1.2386	1.2413	1.2440	1.2466	1.2494	1.2521	53
37	1.2521	1.2548	1.2576	1.2604	1.2632	1.2661	1.2690	52
38	1.2690	1.2719	1.2748	1.2777	1.2807	1.2837	1.2867	51
39	1.2867	1.2898	1.2928	1.2959	1.2990	1.3022	1.3054	50
40	1.3054	1.3086	1.3118	1.3150	1.3183	1.3216	1.3250	49
41	1.3250	1.3283	1.3317	1.3351	1.3386	1.3421	1.3456	48
42	1.3456	1.3491	1.3527	1.3563	1.3599	1.3636	1.3673	47
43	1.3673	1.3710	1.3748	1.3785	1.3824	1.3862	1.3901	46
44	1.3901	1.3940	1.3980	1.4020	1.4060	1.4101	1.4142	45
	60′	50′	40′	30′	20′	10′	0′	Degrees

COSECANT.

SECANT.

Degrees.	0′	10′	20′	30′	40′	50′	60′	
45	1.4142	1.4183	1.4225	1.4267	1.4309	1.4352	1.4395	44
46	1.4395	1.4439	1.4483	1.4527	1.4572	1.4617	1.4662	43
47	1.4662	1.4708	1.4755	1.4801	1.4849	1.4896	1.4944	42
48	1.4944	1.4993	1.5042	1.5091	1.5141	1.5191	1.5242	41
49	1.5242	1.5293	1.5345	1.5397	1.5450	1.5503	1.5557	40
50	1.5557	1.5611	1.5666	1.5721	1.5777	1.5833	1.5890	39
51	1.5890	1.5947	1.6005	1.6063	1.6122	1.6182	1.6242	38
52	1.6242	1.6303	1.6364	1.6426	1.6489	1.6552	1.6616	37
53	1.6616	1.6680	1.6745	1.6811	1.6878	1.6945	1.7013	36
54	1.7013	1.7081	1.7150	1.7220	1.7291	1.7362	1.7434	35
55	1.7434	1.7507	1.7580	1.7655	1.7730	1.7806	1.7882	34
56	1.7882	1.7960	1.8038	1.8118	1.8198	1.8278	1.8360	33
57	1.8360	1.8443	1.8527	1.8611	1.8697	1.8783	1.8870	32
58	1.8870	1.8959	1.9048	1.9138	1.9230	1.9322	1.9416	31
59	1.9416	1.9510	1.9606	1.9702	1.9800	1.9899	2.0000	30
60	2.0000	2.0101	2.0203	2.0307	2.0412	2.0519	2.0626	29
61	2.0626	2.0735	2.0845	2.0957	2.1070	2.1184	2.1300	28
62	2.1300	2.1417	2.1536	2.1656	2.1778	2.1901	2.2026	27
63	2.2026	2.2153	2.2281	2.2411	2.2543	2.2676	2.2811	26
64	2.2811	2.2948	2.3087	2.3228	2.3370	2.3515	2.3662	25
65	2.3662	2.3810	2.3961	2.4114	2.4269	2.4426	2.4585	24
66	2.4585	2.4747	2.4911	2.5078	2.5247	2.5418	2.5593	23
67	2.5593	2.5769	2.5949	2.6131	2.6316	2.6503	2.6694	22
68	2.6694	2.6888	2.7085	2.7285	2.7488	2.7694	2.7904	21
69	2.7904	2.8117	2.8334	2.8554	2.8778	2.9006	2.9238	20
70	2.9238	2.9473	2.9713	2.9957	3.0205	3.0458	3.0715	19
71	3.0715	3.0977	3.1243	3.1515	3.1791	3.2073	3.2360	18
72	3.2360	3.2653	3.2951	3.3255	3.3564	3.3880	3.4203	17
73	3.4203	3.4531	3.4867	3.5209	3.5558	3.5915	3.6279	16
74	3.6279	3.6651	3.7031	3.7419	3.7816	3.8222	3.8637	15
75	3.8637	3.9061	3.9495	3.9939	4.0393	4.0859	4.1335	14
76	4.1335	4.1823	4.2323	4.2836	4.3362	4.3900	4.4454	13
77	4.4454	4.5021	4.5604	4.6202	4.6816	4.7448	4.8097	12
78	4.8097	4.8764	4.9451	5.0158	5.0886	5.1635	5.2408	11
79	5.2408	5.3204	5.4026	5.4874	5.5749	5.6653	5.7587	10
80	5.7587	5.8553	5.9553	6.0588	6.1660	6.2771	6.3924	9
81	6.3924	6.5120	6.6363	6.7654	6.8997	7.0396	7.1852	8
82	7.1852	7.3371	7.4957	7.6612	7.8344	8.0156	8.2055	7
83	8.2055	8.4046	8.6137	8.8336	9.0651	9.3091	9.5667	6
84	9.5667	9.8391	10.127	10.437	10.758	11.104	11.473	5
85	11.473	11.868	12.291	12.745	13.234	13.763	14.335	4
86	14.335	14.957	15.636	16.380	17.198	18.102	19.107	3
87	19.107	20.230	21.493	22.925	24.562	26.450	28.653	2
88	28.653	31.257	34.382	38.201	42.975	49.114	57.298	1
89	57.298	68.757	85.945	114.59	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Degrees.

COSECANT.

DESCRIPTION OF THE TRIGONOMETRICAL SCALE ON THE CALCULATING MACHINE.

In the preceding treatment of triangles, tables of the trigonometrical lines are required for the solutions. When accompanied with one of these *simple calculators*, all those tables are dispensed with, and it is so arranged that it is *not* necessary to notice the function, only operating by the angles themselves, expressed in degrees and minutes. This makes the trigonometrical solutions *so simple* that any one who understands the simple arithmetic will be able to solve the trigonometrical questions.

Explanation.—The inner scale between *sin.* and *cos.*, marked on the arms, is for trigonometrical calculations. The numbers in the circles *sin.* and *cos.* represent the angles in degrees, and the divisions on the arms show the exceeded minutes where the lines intersect the arms. When the arm is set on an angle in the circle *sin.*, the circle *a* shows the length of the *sinus* for that angle. When set on an angle in the circle *cos.*, the circle *a* shows the length of the *cosinus*. By this arrangement, any of the trigonometrical functions can be found.

Example 1.—To find the length of the *tangent* for the angle $C = 54°$.

$$tan. C = \frac{sin. C}{cos. C} = \frac{sin. 54°}{cos. 54°} = 1.3763.$$

Set A on *sin.*54, B on *cos.*54, clamp C. Move the arms until B comes to *zero,* then A shows the length of the *tangent* = 1.3767 circle a.

Example 2.—To find the *secant* for an angle $C = 35°$.

$$sec.35° = \frac{1}{cos.35} = 1.2207.$$

Set A on 1, B on *cos.*35°, clamp C. Move the arms until B comes to *zero,* then A shows the length of the *secant* = 12207.

Example 3. **Fig. 2.**—An inclined plane a is to be built to a height $c = 42$ feet; the angle C is 39°. What will be the length of the plane a?

$$a = \frac{42}{sin.39} = 66.8 \text{ feet.}$$

Set A on 42, B on *sin.*39°, clamp C. Move the arms until B comes to *zero,* then A shows the answer $a = 66.8$ feet.

Example 4.—An inclined plane is to be built for a railroad. The highest point is 264.5 feet above the lower or foot of the plane; the whole plane is 1463 feet long. What is the angle of inclination?

$$sin. C = \frac{2645}{1463} = 10° \; 25'.$$

Set A on 264.5, B on 1463, clamp C. Move the arms until B comes to *zero,* then A shows the angle $C = 10° \; 25'$ on circle *sin.*

Example 5. **Fig. 3.** *Formula* 4.—The side b is 436 feet long; the angle $A = 42° \ 25'$, $B = 21° \ 46'$. What will be the length of the side a?

$$a = \frac{436 \times sin.42° \ 25'}{sin.21° \ 46'} = 793.1 \text{ feet.}$$

Set A on 436, B on $sin.21° \ 46'$, clamp C. Move the arms until B comes to $sin.42° \ 25'$, then the arm A shows the answer 793.1 feet.

Example 6. **Fig. 3.** *Formula* 7.—The angle $B = 46° \ 38'$; $c = 33430$ feet, $b = 83190$ feet. Require the angle $C = ?$

$$sin. C = \frac{33430 \times sin.46° \ 38'}{83190} = 16° \ 59'.$$

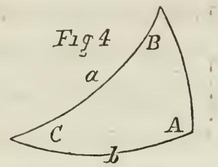
Set A on $sin.46° \ 38'$, B on 83190, clamp C. Move the arms until B comes to 33430, then A shows the answer $C = 16° \ 59'$ on the circle *sin.*

SPHERICAL TRIGONOMETRY.

Spherical trigonometry treats of the triangles which are (or imagined to be) drawn on the surface of a sphere. The sides are arcs of the great circle of the sphere, and measured by the angle of the arc.

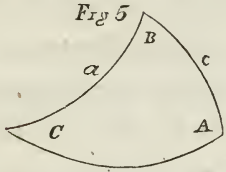
In spherical trigonometry the functions bear quite a different relation to the sides than in plane triangles, which here will be seen.

Fig. 4.—*A right-angled spherical triangle; A* being the right angle.



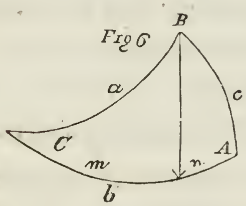
$$sin.a : 1 = sin.c : sin.C$$
$$cos.a : 1 = cot.B : tan.C$$
$$tan.a : 1 = tan.b : cos.C$$
$$sin.c : 1 = tan.b : tan.B$$

Fig. 5.—*An oblique-angled triangle.*



$$sin.a : sin.b = sin.A : sin.B$$
$$sin.b : sin.c = sin.B : sin.C$$

Fig. 6.—*A line drawn from the angle B perpendicular to the side b; the two parts of b are denoted by the letters* m *and* n.



$$cos.a : 1 = cot.m : tan.C$$
$$tan.a : 1 = tan.m : cos.C$$
$$cos.A : tan.c = sin.m : sin.(b - m)$$
$$cos.a : cos.c = cos.m : cos.(b - m)$$
$$tan.a : tan.c = cos.B : cos.m$$

By spherical trigonometry, we ascertain distances and angles on the surface of the earth. It is principally used in navigation; but, even on shore, persons knowing places on the earth, desire to know their exact distance, and in what direction or course the one place is from the other. For that purpose we will prepare the above formulæ. By places being known, we mean their latitudes and longitudes are known; then, in the above formulæ, the quantities A, B, C, and a, b, c, require to be expressed in latitude and longitude.

When the difference in latitude and longitude is given, we obtain, in the difference of longitude, circle planes, which are drawn at a distance from the centre of the earth, equal to the *sine* for the latitude, with a radii $=$ *cosine* for the latitude; then, in the parallel, the distance between two points will be the difference in longitude multiplied by *cosine* for the latitude; but this will not be the shortest distance between the two points; but the arc of a circle plane drawn through the two points, and the centre of the earth, will be the shortest distance.

Letters will denote,

$L =$ difference in longitude between two places, in *time* expressed in degrees of the great circle.

$l =$ latitude in degrees.

$d' =$ distance between two places on the arc of a parallel, in degrees of the great circle.

then $d' = L \cos.l.$

Example 1.—What will be the distance between Boston and Cape Creaux, (South Spain,) their difference in longitude $L = 74° \ 20'$ latitude $l = 42° \ 20'$.

$d' = 74.33 \ cos. \ 42° \ 20' = 54.95°$ multiplied by 60 will be geographical miles 3297, the distance on the parallel.

$d =$ distance on the arc of the great circle in degrees, or shortest distance between two points, then

$$sin.\tfrac{1}{2} \ d = sin.\tfrac{1}{2} \ L \ cos.l \quad . \quad . \quad . \quad . \quad (1)$$

$$\tfrac{1}{2} \ L = \frac{74° \ 20'}{2} = 37° \ 10'$$

$$sin.\tfrac{1}{2} \ d = sin.37° \ 10' \times cos.42° \ 20' = 26° \ 32'$$

$d = 26° \ 32' \times 2 = 53° \ 4'$ multiplied by 60 will be a distance 3184 geographical miles, the shortest distance.

The difference between the two distances will be,

$$\begin{aligned} d' &= 3297 \\ d &= 3184 \\ \hline & \quad 113 \text{ miles.} \end{aligned}$$

When the vessel sails the distance d', she always keeps the same course east or west, but in the distance d, the vessel will always be in a higher latitude than the starting-points, and the course from the first point will be,

$$sin.C = \frac{sin.L \ cos.l}{sin.d} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

$C =$ course in degrees from the meridian, require the course from Boston to Cape Creaux.

$$sin. C = \frac{sin. 74° \ 20' \times cos. 42° \ 20'}{sin. 53° \ 4'} = 62° \ 56'$$

To operate this by the *calculator*, will be the same as before described, viz.:

Set A on $sin. 74° \ 20'$, B on $sin. 53° \ 4'$, clamp C; move the arms, until B comes to $cos. 42° \ 20'$; then A shows the course = $62° \ 56'$ or $5\frac{3}{4}$ points nearly.

When the vessel has sailed half the distance, she is in the highest latitude, which will be found by the formulæ.

$$tan. l' = \frac{tan. l}{cos. \frac{1}{2} L}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$tan. l' = \frac{tan. 42° \ 20'}{cos. 37° \ 4'}, = 48° \ 50'$$

When the two places lie in different latitudes, and their difference in longitude is given, to find the nearest distance and course.

Letters will denote,

l = lower latitude.

l' = highest latitude.

C = course from the latitude l.

C' = course from the latitude l'.

d = shortest distance between l and l' in degrees of the great circle.

L = difference in longitude in degrees

$$tan. m = cot. \ l' \ cos. L.$$

$n = 90 \mp l - m$.

$-\ l$, when l and l' are on one side of the equator.

$+\ l$, when l' is on one side and l on the other.

Then the formulæ for calculating the shortest distances and courses, to and from *any known* points on the earth, will be simply

$$cos.d = \frac{sin.l' \, cos.n}{cos.m}, \quad . \quad . \quad . \quad . \quad (4)$$

$$sin.C = \frac{sin.L \, cos.l'}{sin.d}, \quad . \quad . \quad . \quad . \quad (5)$$

$$sin.C' = \frac{sin.L \, cos.l}{sin.d}, \quad . \quad . \quad . \quad . \quad (6)$$

Example 2.—Require the shortest distance and course from New York to Liverpool?

$$\left.\begin{array}{l} l = 40° \; 42' \text{ N. latitude} \\ \quad 74° \; \text{“ W. longitude} \end{array}\right\} \text{New York,}$$

$$\left.\begin{array}{l} l' = 53° \; 22' \text{ N. latitude} \\ \quad 2° \; 52' \text{ W. longitude} \end{array}\right\} \text{Liverpool,}$$

$L = 71° \;\; 8'$ difference in longitude.

$$tan.m = cot.53° \; 22' \times cos.71° \; 8' = 13° \; 31',$$

$$n = 90 - 13° \; 31' - 40° \; 42' = 35° \; 47',$$

$$cos.d = \frac{sin.53° \; 22' \times cos.35° \; 47'}{cos.13° \; 31'} = 47° \; 58',$$

Shortest distance $= 47 \times 60 + 58 = 2878$ geographical miles.

$$sin.C = \frac{sin.71° \; 8' \times cos.53° \; 22'}{sin.47° \; 58'} = 49° \; 23' = 4\tfrac{3}{8} \text{ points.}$$

Course from New York N. E. $\frac{3}{8}$ E.

Example 3.—What will be the distance and course from San Francisco to Port Jackson, Australia?

$$\left.\begin{array}{l} l' = 37^\circ\ 47'\ \text{N. latitude} \\ \quad 122^\circ\ 21'\ \text{W. longitude} \end{array}\right\}\ \text{San Francisco,}$$

$$\left.\begin{array}{l} l = 33^\circ\ 50'\ \text{S. latitude} \\ \quad 151^\circ\ 25'\ \text{E. longitude} \end{array}\right\}\ \text{Port Jackson,}$$

Difference longitude

$$L = 360 - 122^\circ\ 50' - 151^\circ\ 25' = 86^\circ\ 14'.$$

$$tan.m = cot.37^\circ\ 47' \times cos.86^\circ\ 14' = 4^\circ\ 50',$$

$$n = 90 + 33^\circ\ 50' - 4^\circ\ 50' = 119^\circ,$$

$$cos.d = \frac{sin.37^\circ\ 47' \times cos.119}{cos.4^\circ\ 50'} = 107^\circ\ 20',$$

Shortest distance $= 107 \times 60 + 20 = 6440$ geo. miles.

$$sin.C' = \frac{sin.86^\circ\ 14' \times cos.33^\circ\ 50'}{sin.107^\circ\ 20'} = 60^\circ\ 16' = 5\tfrac{1}{2}\ \text{points.}$$

Course from San Francisco S. W. *b*. W. ½ W. nearly.

When computing those examples by the calculator, there will be no more figuring on paper, than those shown in the print; the machine brings out the answer expressed in degrees and minutes.

Improvements on the Calculating Machine by the addition of two more Scales.

The additional scales do not appear on the accompanying Plate; they are placed one on the inside, and one on the outside of the two scales. The outer one is divided into 90 equal parts around the whole circle, representing the 90 degrees in a quadrant; each of those divisions is divided into 6 equal parts, representing every 10 minutes per degree. This scale is accompanied with a *vernier*, which is divided into 10 equal parts corresponding with 9 divisions on the scale; by this arrangement, every minute on a degree can be distinctly read, as the ten divisions on the *vernier* are in a space about half an inch. The outer edge of the scale is numbered in hours, minutes, and seconds, *in time;* corresponding to degrees and minutes of the circle. The object of this scale is, for adding and subtracting degrees and minutes, and to turn degrees and minutes into *time*.

The inner scale (called the compass) is laid out from the *mariner's compass*, in *points* and fractions thereof, which corresponds with courses, distances, and difference in latitudes, longitude, &c., on the outer scales.

The two additional scales are principally for navigation; and the combination of the four scales will *cover all calculations at sea*, so that navigators will be enabled to make quick and correct calculations, without reference to any tables.

15

NAVIGATION.

To navigate a vessel upon the supposition that the earth is a level plane, on which the *meridians* are drawn *north* and *south* parallel with each other; and the *parallels east* and *west*, at right angles to the former.

Fig. 7.

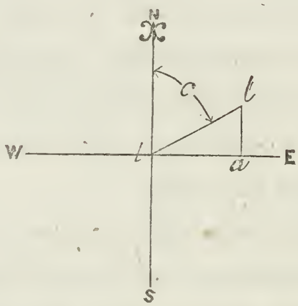


Fig. 7.—The line N. S. represents a *meridian north* and *south;* the line E. W. represents a *parallel east* and *west.*

A ship in l, sailing in the direction l, l', and having reached l', it is required to know her position to the point l, which is measured by the line l, l, and the angle N. $l\,l'$; and imagined by the lines $l\ a$ and $a\ l'$?

These four quantities bear the following names:—

$d = l\,l'$, *distance* from l to l'.

$C =$ N. $l\,l'$, *course* or *points* from the meridian.

$\mathfrak{d} = l\,a$, *departure* or difference in longitude.

$f = a\,l'$, difference in latitude.

When any two of those four quantities are given, the other two can be ascertained by them, which operation will be illustrated by the formulæ.

$$\text{Departure?}\quad \begin{cases} \mathfrak{d} = d \, sin. C & \ldots\ldots \quad (1) \\ \mathfrak{d} = f \, tan. C & \ldots\ldots \quad (2) \end{cases}$$

$$\text{Diff. latitude?}\quad \begin{cases} f = d \, cos. C & \ldots\ldots \quad (3) \\ f = \mathfrak{d} \, tan. C & \ldots\ldots \quad (4) \end{cases}$$

$$\text{Distance?}\quad \begin{cases} d = \dfrac{\mathfrak{d}}{sin. C}, & \ldots\ldots \quad (5) \\[2ex] d = \dfrac{f}{cos. C}, & \ldots\ldots \quad (6) \end{cases}$$

$$\text{Course?}\quad \begin{cases} cos. C = \dfrac{f}{d}, & \ldots\ldots \quad (7) \\[2ex] sin. C = \dfrac{\mathfrak{d}}{d}, & \ldots\ldots \quad (8) \\[2ex] tan. C = \dfrac{\mathfrak{d}}{f}, & \ldots\ldots \quad (9) \end{cases}$$

When the *course* is expressed in points from the meridian, we will change the characters for C, to distinguish them from the expression of degrees; namely,

$$C \text{ in points} \left\{ \begin{array}{l} lat. C = cos. C \\ dep. C = sin. C \end{array} \right\} C \text{ in degrees.}$$

d, $\mathfrak{d}$, and l, are geographical miles.

The characters represent

lat. = difference in latitude;

dep. = departure.

These characters are marked on the arms, over the *compass* on the machine.

While the vessel is running from l to l', the distance is measured by the *log* and *time;* and the *course* is measured by the *compass*, commonly expressed in points.

Example 1.—A vessel sails *east north-east* (6 points) 236 miles. Require her departure and difference in latitude?

Set A on 236, B on *zero*, clamp C. Move the arms until *lat.B* comes to 6 points; then A shows the difference in latitude = 90.3 miles.

Set *dep.B* on 6 points, then A shows the departure = 218 miles.

Example 2.—A ship sails between *north* and *west* 173 miles, and found her difference in *latitude* was 74 miles. Require her *course* and *departure?*

Set A on the distance 173 miles, B on 74, clamp C. Move the arm until A comes to *zero;* then *lat.B* shows the course = $5\frac{3}{4}$ points.

Set A on 173, B on *zero*, clamp C. Move the arms until *dep.B* comes to $5\frac{3}{4}$ points; then A shows the departure = 156 miles.

In the same manner, any question in plane sailing is computed. The supposition that the earth is a level plane will suffice when navigating a ship near the equator; but, in higher latitudes and long distances, it will be necessary to partly relinquish this supposition, to find the difference in longitude of the meridians in degrees or in *time* at the equator.

At the equator, there are 60 geographical miles per degree, but in higher latitudes there are so many miles *less* in a degree as the *cosine* for the latitude is *less* than the radii. Say the departure $\mathfrak{d}$ to be the difference in longitude in miles at the latitude l, and

L the difference in longitude in degrees at the equator, we have

$$L : \mathfrak{d} = 1 : 60 \; cos.l,$$

which will be in the form of equations.

$$\textit{Departure?} \quad \left\{ \begin{array}{ll} \mathfrak{d} = 60 \; cos.l \; L, & \quad . \; . \; . \; . \quad (10) \\ \mathfrak{d} = \sqrt{d^2 - f^2}, & \quad . \; . \; . \; . \quad (11) \end{array} \right.$$

$$\textit{Diff. latitude?} \quad \left\{ \begin{array}{ll} f = \dfrac{60 \; cos.l \; L}{tan.C}, & \quad . \; . \; . \; . \quad (12) \\ f = \sqrt{d^2 - \mathfrak{d}^2} & \quad . \; . \; . \; . \; . \quad (13) \end{array} \right.$$

$$\textit{Distance?} \quad \left\{ \begin{array}{ll} d = \dfrac{60 \; cos.l \; L}{sin.C}, & \quad . \; . \; . \; . \quad (14) \\ d = \sqrt{\mathfrak{d}^2 + f^2}, & \quad . \; . \; . \; . \; . \quad (15) \end{array} \right.$$

$$\textit{Course?} \quad \left\{ \begin{array}{ll} sin.C = \dfrac{60 \; cos.l \; L}{d}, & \quad . \; . \quad (16) \\ tan.C = \dfrac{60 \; cos.l \; L}{f}, & \quad . \; . \quad (17) \end{array} \right.$$

$$\textit{Latitude?} \quad \left\{ \begin{array}{ll} cos.l = \dfrac{\mathfrak{d}}{60 \; L}, & \quad . \; . \; . \; . \; . \quad (18) \\ cos.l = \dfrac{d.sin.C}{60 \; L}, & \quad . \; . \; . \; . \quad (19) \\ cos.l = \dfrac{f \; tan.C}{60 \; L}, & \quad . \; . \; . \; . \quad (20) \end{array} \right.$$

$$\textit{Diff. longitude?} \quad \left\{ \begin{array}{ll} L = \dfrac{\mathfrak{d}}{60 \; cos.l}, & \quad . \; . \; . \; . \; . \quad (21) \\ L = \dfrac{d.sin.C}{60 \; cos.l}, & \quad . \; . \; . \; . \; . \quad (22) \\ L = \dfrac{f \; tan.C}{60 \; cos.l}, & \quad . \; . \; . \; . \; . \quad (23) \end{array} \right.$$

Now for any question of *departure, difference in latitude, distance, course, latitude, difference in longitude,* there is one formula which contains the given quantities and gives the answer.

Example 3.—A ship sails in north latitude in a course E. S. E. $\frac{3}{4}$ E. = $6\frac{3}{4}$ points. In a distance of 132 miles, she made a difference in longitude of 3° 34′. What *latitude* is she in?

This question will be answered by the formula (19), which contains the given quantities *course, distance,* and *difference in longitude.* First multiply 60 by 3°, and add 34′ = 214′ miles.

$$cos.l = \frac{132 \times dep.\ 6\frac{3}{4}}{214} = 53^\circ\ 15', \text{ the latitude.}$$

Set A on 214, B on 132, clamp C. Move A to *dep.* $6\frac{3}{4}$ points; the $cos.B$ shows the latitude = 53° 15′.

In the same simple manner, all the questions are solved by the machine, which gives a positive answer instantly.

Persons who do not possess this machine will still find the formulæ convenient. Engineers on steamboats often wish to know their positions at sea, for which purpose the accompanying table is inserted.

North.	South.	Points.	Degrees.	sin. C. dep. C.	cos. C. lat. C.	tan. C. $\frac{dep.}{lat.}$ C.
N.	S.	$\frac{1}{4}$	2° 49′	.0491	.9988	.0492
		$\frac{1}{2}$	5 37	.0979	.9952	.0983
		$\frac{3}{4}$	8 26	.1544	.9880	.1982
N. by E. and N. by W.	S. by E. and S. by W.	1	11 15	.1936	.9811	.1989
		$1\frac{1}{4}$	14 4	.2430	.9700	.2505
		$1\frac{1}{2}$	16 52	.2901	.9570	.3032
		$1\frac{3}{4}$	19 41	.3368	.9416	.3577
N. N. E. and N. N. W.	S. S. E. and S. S. W.	2	22 30	.3827	.9239	.4142
		$2\frac{1}{4}$	25 19	.4276	.9039	.4730
		$2\frac{1}{2}$	28 7	.4713	.8820	.5343
		$2\frac{3}{4}$	30 56	.5140	.8577	.5993
N. E. by N. and N. W. by N.	S. E. by S. and S. W. by S.	3	33 45	.5555	.8314	.6883
		$3\frac{1}{4}$	36 44	.5981	.8014	.7463
		$3\frac{1}{2}$	39 22	.6343	.7731	.8204
		$3\frac{3}{4}$	42 11	.6715	.7410	.9062
N. E. and N. W.	S. E. and S. W.	4	45 0	.7071	.7071	1.000
		$4\frac{1}{4}$	47 49	.7410	.6715	1.103
		$4\frac{1}{2}$	50 37	.7731	.6345	1.218
		$4\frac{3}{4}$	53 26	.8014	.5981	1.348
N. E. by E. and N. W. by W.	S. E. by E. and S. W. by W.	5	56 15	.8314	.5555	1.496
		$5\frac{1}{4}$	59 4	.8577	.5140	1.668
		$5\frac{1}{2}$	61 52	.8820	.4713	1.870
		$5\frac{3}{4}$	64 41	.9039	.4276	2.114
E. N. E. and W. N. W.	E. S. E. and W. S. W.	6	67 30	.9239	.3827	2.414
		$6\frac{1}{4}$	70 19	.9416	.3368	2.795
		$6\frac{1}{2}$	73 7	.9570	.2901	3.295
		$6\frac{3}{4}$	75 56	.9700	.2430	3.991
E. by N. and W. by N.	E. by S. and W. by S.	7	78 45	.9811	.1936	5.027
		$7\frac{1}{4}$	81 34	.9880	.1544	6.744
		$7\frac{1}{2}$	84 22	.9952	.0979	11.14
		$7\frac{3}{4}$	87 11	.9988	.0491	20.32
East or west . . .		8	90°	1.000	0.000	∞

Example 4.—A ship sails from Sandy Hook (New York), latitude 40° 27′, in a course S. E. by S. ½ E. = 3½ points, until her latitude is 32° 54′. Require her distance and difference in longitude from Sandy Hook?

From	43° 27′
Subtract . . .	32° 54′
Difference in latitude $f =$	7° 33′ × 60 = 453 miles.

From the formula (6) we have the distance

$$d = \frac{453}{cos.3\frac{1}{2}} = \frac{453}{0.7731} = 585.5 \text{ miles.}$$

From the formula (23) we have

$$L = \frac{453 \times tan.3\frac{1}{2}}{60 \times cos.3254} = \frac{453 \times 0.8204}{60 \times 0.84} = 7.36 = 7° \; 21.6',$$

the difference in longitude.

Longitude of Sandy Hook . . .	74° 00.5′
Subtract	7° 21.6′
Longitude in	66° 39′

From this point, require the course and distance to Cape Florida?

Latitude . .	$l =$ 25° 41′	Cape Florida
Longitude . .	= 80° 05′	
Subtract . .	66° 39′	
Difference in longitude $L =$	13° 26′	
From . . .	32° 54′	
Subtract . .	25° 41′	
Difference in latitude $f =$	7° 13′ × 60 = 433 miles.	

From the formula (17) we have

$$tan. C = \frac{60 \times cos. 25° 41' \times 13° 26'}{433} = 1.676 = 5\tfrac{1}{4} \text{ points,}$$

or course = S. W. by W. ¼ W.

From the formula (6) we have

$$\text{distance } d = \frac{433}{cos. 5\tfrac{1}{4}} = \frac{433}{0.514} = 841 \text{ miles.}$$

When the *course* and *distance* are required very accurate, calculate them from the formulæ on p. 215.

To find the Trigonometrical lines for any minute, by the accompanying tables.

Example 1.—Find the length of *sin.*35° 34′,

from *sin.*35° 40′ =	0.58306
subtract *sin.*35° 30′ =	0.58070
proportional part .	236
multiply by . .	0.4
	94.4
add . . .	58070

*sin.*35° 34′ = 0.581644, the answer.

Example 2.—Find the length of the *tan.*68° 47′?

from *tan.*68° 50′ =	2.5826
subtract *tan.*68° 40′ =	2.5604
proportional part .	222
multiply by .	0.7
	154.4
add . . .	2.5604

*tan.*68° 47′ = 2.57584, the answer.

APPENDIX.

A few simple rules to calculate the time and position on the earth, from the motion of the heavenly bodies.

Letters will denote,

A = *meridian altitude* of the sun or any other heavenly body.

a = any other *observed altitude* of the same. (A and a = correct centre altitudes.)

D = declination of the sun, or any other heavenly body.

l = *latitude* in the place of observation.

L = longitude between the meridian in the place of observation, and the meridian where the heavenly body passes under the observation.

v = the angle at which the earth rotates, before or after six o'clock, when the heavenly bodies set or rise, viewed from the latitude l.

t = apparent *solar time* in hours.

t' = apparent *moon time* in hours.

x = time in minutes, when the moon passes from one meridian to another, and the longitude between the two meridians is L.

$$l = 90 - A \pm D, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$A = 90 - l \pm D, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$D = \mp 90 \pm A \pm l, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$sin.v = tan.l \ tan.D \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$cos.L = \frac{sin.a \ (1 \pm sin.v)}{sin.A} \mp sin.v \quad . \quad . \quad (5)$$

$$tan.l = \frac{sin.a' \ cos.L - sin.a \ cos.L'}{tan.D \ (\mp sin.a \pm sin.a')} \quad . \quad . \quad (6)$$

$$tan.D = \frac{sin.a' \ cos.L - sin.a \ cos.L'}{tan.l \ (\mp sin.a \pm sin.a')} \quad . \quad . \quad (7)$$

$$t = \frac{L}{15} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$t' = 0.965 \ t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$x = 0.14 \ L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Where the quantities have the double signs *plus* and *minus*, use the top signs when the latitude and declination are of equal names; use the bottom signs when the latitude and declination are of different names.

Example 1. *Formula* 1.—On the 21st day of October, 1852, the meridian altitude of the sun's lower limb was observed to be 36° 27′. Require the latitude?

Observed altitude		36°	27′
Correction, semid. parallax, &c., add		0	10′
Correct centre altitude . .	$A =$	46	37′
Subtracted from		90	
True zenith distance . . .		43	23
Sun's declination, subtract .	$D =$	11	0′ S
Latitude in	$l =$	32	23 N

Example 2. *Formula* 4.—What time does the sun set and rise, on the 27th day of May, 1853, in latitude 43° 19′?

$$\text{The sun's declination} \begin{cases} \text{in the morning } D = 21° \; 22' \\ \text{in the afternoon } D = 21° \; 30' \end{cases}$$

$$sin.v = tan.43° \; 19' \times tan.21° \; 22' = 0.3689 = 21° \; 20',$$

$$t = \frac{90 - 21° \; 20'}{15} = 4^h \; 34' \; 40'' \text{ o'clock in the morn.}$$

$$sin.v = tan.43° \; 19' \times tan.21° \; 30' = 0.3714 = 21° \; 48'$$

$$t = \frac{90 + 21° \; 48'}{15} = 7^h \; 27' \; 12'' \text{ o'clock in the evening.}$$

Example 3. *Formula* 5.—To find the *apparent time*. In April 17, 1853, the correct *altitude* of the *sun* was observed in the afternoon to be $a = 31° \; 31'$ in the latitude $l = 38° \; 47' \; N$; the sun's declination, at the time of observation, was $D = 10° \; 37' \; N$.

Require the apparent time of observation?

$$sin.v = tan.38° \; 47' \times tan.10° \; 37' = 0.1506,$$

$$A = 90 - 38° \; 47' + 10° \; 37' = 61° \; 50'$$

$$cos.L = \frac{sin.31° \; 31' \; (1 + 0.1506)}{sin.61° \; 50'} - 0.1506 = 0.5313$$

$$= 57° \; 54',$$

$$t = \frac{57° \; 54'}{15} = 3^h \; 51' \; 36'', \text{ the apparent time.}$$

When it is a forenoon observation, t will be

$$t = \frac{90 + L}{15}$$

When the difference in longitude is to be found by the *apparent time*, and the time shown by the chronometer, it will be necessary to notice the difference between the *apparent* and *mean solar time*.

The difference between *apparent time* and *mean time* is attributable to the irregularity of the earth's rotation around its axis, which causes a number of days, "say 100," in one part of the year, to be half an hour longer than 100 days in another part of the year, when a day is the time between the sun's passage over one meridian; this time is called the *apparent time.*

Mean time is the time shown by a chronometer, and is always uniform, so that 100 days in one part of the year are always *equal* to 100 days in any other part of the year. The difference between the *apparent* and *mean time* is called the *equation of time*, and always found in the nautical almanac, where it is noted if it is to be added *to* or subtracted *from* the apparent time.

Example 4.—In connection with the preceding example, suppose the chronometer shows the mean time at Greenwich $7^h\ 27'\ 55''$ at the time of observation. Require the longitude from Greenwich?

Apparent time		$3^h\ 51'\ 36''$
Equation of time, subtract		$30''$
Mean time	subtract	$3^h\ 51'\ 6''$
	from	$7^h\ 27'\ 55''$
		$3^h\ 36'\ 49''$
Multiply by		15
West longitude		$54^\circ\ 12'\ 15''$

When the longitude is to be found by an altitude *a* of the moon, the *equation* of time need not be noticed, because it is contained in the time when the moon passes the meridian at Greenwich.

Example 5.—On the 25th day of September, 1852, in north latitude 22° 35′, and west longitude about 53° 9′, at $7^h\,15'$ o'clock by watch, was taken an altitude

Of the moon's lower limb . . .	33° 42′
Correction semd. parlx. refn. add .	45′
☽ correct centre altitude . .	$a =$ 34° 27′
☽ declination corrected . .	$D =$ 13° 52′ 43″ S
Passes the meridian at Greenwich .	$10^h\ 21'.6$
Correction add . $x = 53 \times 0.14 =$	$0^h\ \ 7'.4$
☽ passes the meridian	$10^h\ 29'$

Require the *mean time* and *longitude* from Greenwich?

$$sin.v = tan.22°\ 35' \times tan.13°\ 52'\ 43'' = 0.10275,$$

$$A = 90 - 13°\ 52'\ 43'' - 22°\ 35' = 53°\ 32'\ 17''$$

$$cos.L = \frac{sin.34°\ 27'\ (1-0.10275)}{sin.53°\ 32'\ 17''} + 0.10275 = 0.73385$$

$= 42°\ 47'$,

$$t' = \frac{42°\ 47'}{15} = 2^h\ 51'\ 8'' \text{ apparent moon time.}$$

Divided by 0.965 will be $t = 2^h\ 57'\ 20''$.

☽ passes the meridian at . . .	$10^h\ 29'$
Subtract . . . . . . $t =$	$2^h\ 57'\ 20''$
Mean time of observation . . .	$7^h\ 31'\ 40''$
Watch too slow	$16'\ 40''$
Time of observation by *chronometer* .	$11^h\ 13'\ 15''$
Subtract *mean time* of obs. . . .	$7^h\ 31'\ 40''$
	$3^h\ 41'\ 35''$
Multiply by	15
West longitude	55° 23′ 45″

The formula (6) is for calculating the *latitude* from two altitudes of the sun or any other heavenly body, when the *times* of observation are known. If this formula is to be used for the *moon,* it is necessary to notice the difference in the declination at the *times* of observation; then the formulæ will appear as

$$tan.l = \frac{sin.a' \; cos.L - sin.a \; L'}{\mp tan.D \, sin.a \pm tan.D' \; sin.a'}$$

in which D and D' are declinations at the altitudes a and a'.

These calculations are computed very rapidly by the CALCULATOR, owing to the trigonometrical functions being instantly at hand, and *no tables* but the nautical almanac being required.

THE END.

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